$\boxed{28}$ En $[1,3]$ $y=\frac{x^2}{2}\geq 0$, $y=\frac{4}{x}\geq 0$

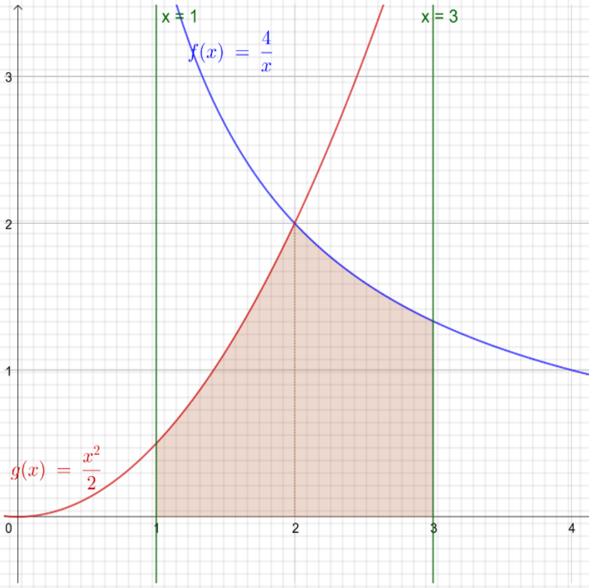
a) Intersecamos $y=\frac{x^2}{2}$ e $y=\frac{4}{x}$

$$\begin{cases} y=\frac{x^2}{2} \\ y=\frac{4}{x}\end{cases} \Longleftrightarrow \frac{x^2}{2}=\frac{4}{x} \Longleftrightarrow x^3=8 \Longleftrightarrow x=2$$

Determinamos la posición relativa de las curvas en $[1,3]$:

$$\left.\begin{array}{l} x=\frac{3}{2}\in[1,2] \\ y=\frac{(3/2)^2}{2}=\frac{9}{8} \\ y=\frac{4}{3/2}=\frac{8}{3}\end{array}\right| \; y=\frac{4}{x} \text{ está encima de } y=\frac{x^2}{2}$$

$$\left.\begin{array}{l} x=\frac{5}{2} \\ y=\frac{(5/2)^2}{2}=\frac{25}{8} \\ y=\frac{4}{5/2}=\frac{8}{5}\end{array}\right| \Longrightarrow y=\frac{x^2}{2} \text{ está encima de } y=\frac{4}{x}$$



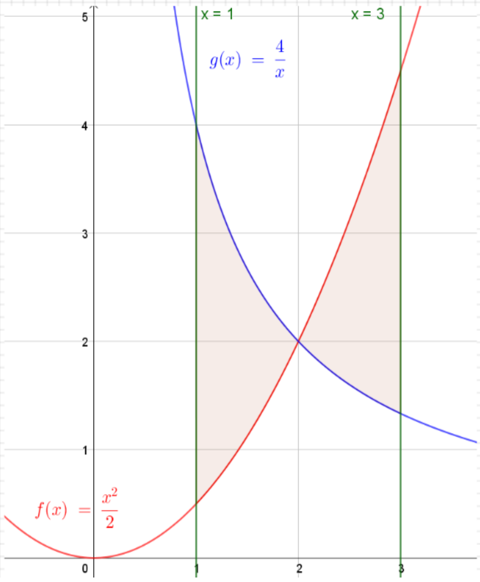
$$\text{ÁREA}=\int_1^2 \frac{x^2}{2}dx+\int_2^3\frac{4}{x}dx=\left[\frac{x^3}{6}\right]_1^2+\left[4\ln x\right]_2^3=\frac{8}{6}-\frac{1}{6}+4\ln 3-4\ln 2=\frac{7}{6}+4\ln\left(\frac{3}{2}\right)$$

b) $$\text{ÁREA}=\int_1^2 \frac{4}{x}-\frac{x^2}{2}dx+\int_2^3\frac{x^2}{2}-\frac{4}{x}dx=$$

$$=\left[4\ln x-\frac{x^3}{6}\right]_1^2+\left[\frac{x^3}{6}-4\ln x\right]_2^3=$$

$$=4\ln 2-\frac{8}{6}-\left(0-\frac{1}{6}\right)+\frac{27}{6}-4\ln 3-\left(\frac{8}{6}-4\ln 2\right)=$$

$$=8\ln 2-4\ln 3+2$$



$\boxed{28}$ c) $f(x) = \dfrac{-4x}{(1+x^2)^2} \qquad g(x) = -x$

Intersecamos las curvas

$$\begin{cases} y = -\dfrac{4x}{(1+x^2)^2} \\ y = -x \end{cases} \Longleftrightarrow \frac{-4x}{(1+x^2)^2} = -x \Longleftrightarrow 4x = x(1+x^2)^2 \Longleftrightarrow$$

$$\Longleftrightarrow \begin{cases} x = 0 \\ (x^2+1)^2 = 4 \end{cases} \Longleftrightarrow x^2+1 = \pm 2 \Longrightarrow \begin{cases} x^2+1 = -2 \quad (\text{sin solución} \\ x^2+1 = 2 \Longrightarrow x = \pm 1 \end{cases}$$

Los puntos de corte entre ambas curvas son $(1,-1), (0,0)$, y $(-1,1)$

Entre $x=-1$ y $x=0$:

$$\left.\begin{array}{l} f\left(-\frac{1}{2}\right) = \dfrac{2}{\left(1+\frac{1}{4}\right)^2} = \dfrac{2}{\frac{25}{16}} = \dfrac{32}{25} \\ g\left(-\frac{1}{2}\right) = \frac{1}{2} \end{array}\right\} \Longrightarrow f \geq g$$

Entre $x=0$ y $x=1$:

$$\left.\begin{array}{l} f\left(\frac{1}{2}\right) = -\frac{32}{25} \\ g\left(\frac{1}{2}\right) = -\frac{1}{2} \end{array}\right\} \Longrightarrow g \geq f(x)$$

$$\text{ÁREA} = \int_{-1}^{0} f-g + \int_{0}^{1} g-f = \int_{-1}^{0} -\frac{4x}{(1+x^2)^2} + x \, dx + \int_{0}^{1} -x + \frac{4x}{(1+x^2)^2} \, dx =$$

$$= \left[\frac{2}{1+x^2} + \frac{x^2}{2} \right]_{-1}^{0} + \left[-\frac{x^2}{2} - \frac{2}{1+x^2} \right]_{0}^{1} = 2 + 0 - \left(1+\frac{1}{2}\right) - \frac{1}{2} - 1 - \left(-0-2\right) =$$

$$= 2 - 1 - \frac{1}{2} - \frac{1}{2} - 1 + 2 = \boxed{1}$$