$\boxed{24}$ $f(x) = |x^2 - 2x| = \begin{cases} x^2 - 2x & \text{si } x \leq 0 \text{ o } x \geq 2 \\ -x^2 + 2x & \text{si } 0 \leq x \leq 2 \end{cases}$

$x^2 - 2x = 0 \Longleftrightarrow x(x-2) = 0 \Longleftrightarrow \begin{cases} x = 0 \\ x = 2 \end{cases}$

SIGNO $x^2 - 2x$: $+$ (0) $-$ (2) $+$

Representamos $y = x^2 - 2x$:

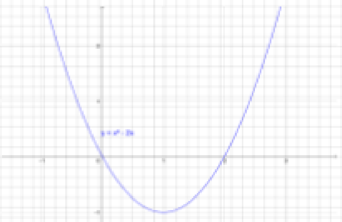
Puntos de corte con los ejes: $(0,0)$, $(2,0)$

$y' = 2x - 2 \qquad y' = 0 \Longleftrightarrow 2x - 2 = 0 \Longleftrightarrow x = 1$

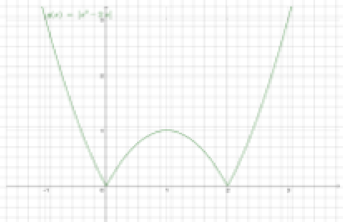
$x = 1 \Rightarrow y = 1^2 - 2.1 = -1 \Rightarrow V = (1,-1)$

y crece en $(1, +\infty)$ y decrece en $(-\infty, 1)$

$y'' = 2 > 0 \Rightarrow y = \text{convexa}$



$\Longrightarrow$



Representamos $y = 6x - x^2$:

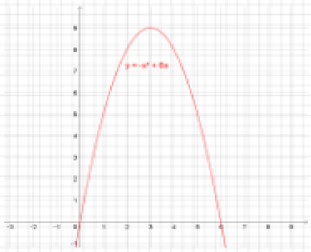
Puntos de corte con los ejes: $(0,0)$, $(6,0)$

$y' = 6 - 2x \Rightarrow y' = 0 \Longleftrightarrow 6 - 2x = 0 \Longleftrightarrow x = 3$

Vértice $(3, 9)$

y crece en $(-\infty, 3)$, y decrece en $(3, +\infty)$

$y'' = -2 \Longrightarrow y$ es cóncava



Intersección entre $y = |x^2 - 2x|$, $y = 6x - x^2$

$|x^2 - 2x| = 6x - x^2 \Longleftrightarrow \begin{cases} x^2 - 2x = 6x - x^2 \Rightarrow 2x^2 - 8x = 0 \Rightarrow \begin{matrix} x = 0 \rightarrow (0,0) \\ x = 4 \rightarrow (4, 8) \end{matrix} \\ x^2 - 2x = -6x + x^2 \Rightarrow 4x = 0 \rightarrow x = 0 \end{cases}$

Posición de ambas curvas:

$x = 2 \in [0,4] \Rightarrow \begin{matrix} |2^2 - 2.2| = 0 \\ 6.2 - 2^2 = 8 \end{matrix} \Big\} \Rightarrow y = 6x - x^2$ está por encima

$$\text{ÁREA} = \int_0^4 6x - x^2 - |x^2 - 2x|\,dx = \int_0^2 6x - x^2 - (-x^2 + 2x)\,dx + \int_2^4 6x - x^2 - (x^2 - 2x)\,dx =$$

$$= \int_0^2 4x\,dx + \int_2^4 -2x^2 + 8x\,dx = \left[2x^2\right]_0^2 + \left[-\frac{2}{3}x^3 + 4x^2\right]_2^4 = 2.4 - 0 - \frac{2}{3}.4^3 + 4^3 - \left(-\frac{2}{3}.2^3 + 4.2^2\right) =$$

$$= 8 - \frac{2}{3}.64 + 64 + \frac{2}{3}.8 - 16 = \frac{56}{3}\ u^2$$