$\boxed{22}$ $f(x) = \operatorname{sen} x$

$g(x) = \operatorname{sen}(2x)$

1º) Calculamos los puntos de intersección entre las curvas:

$$\begin{cases} y = \operatorname{sen} x \\ y = \operatorname{sen}(2x) \end{cases} \Longleftrightarrow \operatorname{sen} x = \operatorname{sen}(2x) \Longleftrightarrow$$

$$\Longleftrightarrow \operatorname{sen} x = 2 \operatorname{sen} x \cos x \Longleftrightarrow \begin{matrix} \operatorname{sen} x = 0 \\ 1 = 2\cos x \end{matrix} \Longleftrightarrow$$

$$\Longleftrightarrow \begin{matrix} \operatorname{sen} x = 0 \\ \cos x = \frac{1}{2} \end{matrix} \Longleftrightarrow \begin{matrix} x = 0 + 2k\pi,\ x = \pi + 2k\pi,\ k \in \mathbb{Z} \\ x = \frac{\pi}{3} + 2k\pi,\ x = -\frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \end{matrix}$$

En $[0, \pi]$, los puntos de intersección son en $x = 0$, $x = \frac{\pi}{3}$, y $x = \pi$

El intervalo de integración queda dividido en dos partes: $[0, \frac{\pi}{3}]$ y $[\frac{\pi}{3}, \pi]$

En $[0, \frac{\pi}{3}]$:

$$\left.\begin{matrix} \frac{\pi}{6} \in [0, \pi/3] \\ f\left(\frac{\pi}{6}\right) = \operatorname{sen}\left(\frac{\pi}{6}\right) = \frac{1}{2} \\ g\left(\frac{\pi}{6}\right) = \operatorname{sen}\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \end{matrix}\right\} \Rightarrow g \geq f$$

En $[\frac{\pi}{3}, \pi]$:

$$\left.\begin{matrix} \frac{\pi}{2} \in [\frac{\pi}{3}, \pi] \\ f\left(\frac{\pi}{2}\right) = \operatorname{sen}\left(\frac{\pi}{2}\right) = 1 \\ g\left(\frac{\pi}{2}\right) = \operatorname{sen} \pi = 0 \end{matrix}\right\} \Rightarrow f \geq g$$

Por tanto:

$$\text{ÁREA} = \int_0^{\pi/3} \operatorname{sen}(2x) - \operatorname{sen} x \, dx + \int_{\pi/3}^{\pi} \operatorname{sen} x - \operatorname{sen}(2x) \, dx = \left[-\frac{\cos(2x)}{2} + \cos x \right]_0^{\pi/3} +$$

$$+ \left[-\cos x + \frac{\cos(2x)}{2} \right]_{\frac{\pi}{3}}^{\pi} = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} - \left(-\frac{1}{2} + 1 \right) + 1 + \frac{1}{2} - \left(-\frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2} \right) =$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2} - \cancel{1} \cancel{+1} + \frac{1}{2} + \frac{1}{2} + \frac{1}{4} = \frac{5}{2} \ u^2$$

$g(x) = sen(2x)$
$f(x) = sen(x)$
1
0.75
0.5
0.25
0
−0.25
−0.5
−0.75
−1
π / 6
π / 3
π / 2
2π / 3
5π / 6
π