$\boxed{13}$ a) $\displaystyle\int \frac{3x}{x^2+1}dx = 3\cdot\frac{1}{2}\int\frac{2x}{x^2+1}dx = \frac{3}{2}\ln(x^2+1)+C$

b) $\displaystyle\int \frac{5x}{2x^2+3}dx = \frac{5}{4}\int\frac{4x}{2x^2+3} = \frac{5}{4}\ln(2x^2+3)+C$

c) $\displaystyle\int \frac{1}{x^2-2x+1}dx = \int\frac{dx}{(x-1)^2} = \frac{(x-1)^{-2+1}}{-2+1} = -\frac{1}{x-1}+C$

d) $\displaystyle\int \frac{3x-1}{x^3-x}dx = \ln|x^3-x|+C$

e) $\displaystyle\int \frac{1}{5-x^2}dx = \frac{1}{2\sqrt5}\int\frac{1}{\sqrt5-x}dx + \frac{1}{2\sqrt5}\int\frac{1}{\sqrt5+x}dx = -\frac{1}{2\sqrt5}\ln|\sqrt5-x| + \frac{1}{2\sqrt5}\ln|\sqrt5+x|+C$

$\displaystyle\frac{1}{5-x^2} = \frac{A}{\sqrt5-x}+\frac{B}{\sqrt5+x} \Rightarrow 1 = A(\sqrt5+x)+B(\sqrt5-x)$

$x=\sqrt5 \Rightarrow A = \frac{1}{2\sqrt5}$

$x=-\sqrt5 \Rightarrow B = \frac{1}{2\sqrt5}$

f) $\displaystyle\int \frac{1-4x}{2x^3-x^2-x}$

$2x^3-x^2-x = x(2x^2-x-1)$

$2x^2-x-1=0 \Rightarrow x = \frac{1\pm\sqrt{1+8}}{4} = \frac{1\pm3}{4} = \begin{cases}1\\ -\frac12\end{cases}$

$2x^2-x-1 = 2(x+1)(x-\frac12) = (x+1)(2x-1)$

$2x^3-x^2-x = x(x+1)(2x-1)$

$\displaystyle\frac{1-4x}{2x^3-x^2-x} = \frac{A}{x}+\frac{B}{x+1}+\frac{C}{2x-1}$

$1-4x = A(x+1)(2x-1)+Bx(2x-1)+Cx(x+1)$

$x=0 \Rightarrow 1=-A \Rightarrow A=-1$

$x=-1 \Rightarrow 5=3B \Rightarrow B=\frac53$

$x=\frac12 \Rightarrow -1=\frac34C \Rightarrow C=-\frac43$

$\displaystyle\int\frac{1-4x}{2x^3-x^2-x}dx = -1\int\frac{dx}{x}+\frac53\int\frac{dx}{x+1}-\frac43\cdot\frac12\int\frac{2}{2x-1}dx =$

$= -\ln|x|+\frac53\ln|x+1|-\frac23\ln|2x-1|+C$

$e^{3x}=e^{2x}\cdot e^{x}$

$\boxed{13}$ g) $\int \frac{e^{3x}}{e^{2x}-3e^{x}+2}\,dx = \int \frac{t^2\,dt}{t^2-3t+2} = \int \frac{t^2-3t+2+3t-2}{t^2-3t+2}\,dt = \int 1+\frac{3t-2}{t^2-3t+2}\,dt$

$e^x=t \Rightarrow e^x dx = dt$

$\frac{3t-2}{t^2-3t+2} = \frac{A}{t-2}+\frac{B}{t-1} \Rightarrow 3t-2 = A(t-1)+B(t-2)$

$t=1 \Rightarrow \quad 1=-B \Rightarrow B=-1$

$t=2 \Rightarrow \quad 4=A$

$\int 1+\frac{3t-2}{t^2-3t+2}\,dt = \int 1+\frac{4}{t-2}+\frac{-1}{t-1}\,dt = t+4\ln|t-2|-\ln|t-1|+C =$

$= e^x+4\ln|e^x-2|-\ln|e^x-1|+C$

h) $\int (x^2-2x)e^{-x}\,dx = -(x^2-2x)e^{-x}+\int(2x-2)e^{-x}\,dx = -(x^2-2x)e^{-x}-(2x-2)e^{-x}+\int 2e^{-x}dx=$

$u=x^2-2x \rightarrow du=2x-2\,dx$
$dv=e^{-x}dx \rightarrow v=-e^{-x}$

$u=2x-2 \rightarrow du=2dx$
$dv=e^{-x}dx \rightarrow v=-e^{-x}$

$= -(x^2-2x)e^{-x}-(2x-2)e^{-x}-2e^{-x}+C = -x^2e^{-x}+C$

i) $\frac{1}{2}\int \frac{2x}{x^4+16}\,dx = \frac{1}{2}\int\frac{dt}{t^2+16} = \frac{1}{2}\int \frac{1}{16\left(\left(\frac{t}{4}\right)^2+1\right)}dt = \frac{1}{2}\cdot\frac{1}{16}\cdot 4\int\frac{\frac{1}{4}\,dt}{1+\left(\frac{t}{4}\right)^2} = \frac{1}{8}\operatorname{arctg}\left(\frac{t}{4}\right) = \frac{1}{8}\operatorname{arctg}\left(\frac{x^2}{4}\right)+C$

$x^4=(x^2)^2$

$t=x^2 \Rightarrow dt=2x\,dx$

j) $\int \frac{4x^3+2x-1}{2x+1}\,dx$

$$\begin{array}{l|l}
4x^3+2x-1 & 2x+1 \\
\hline
-4x^3-2x^2 & 2x^2-x+\frac{3}{2} \\
\quad -2x^2+2x-1 & \\
\quad\ 2x^2+x & \\
\qquad 3x-1 & \\
\qquad -3x-3/2 & \\
\qquad -5/2 &
\end{array}$$

$\frac{4x^3+2x-1}{2x+1} = 2x^2-x+\frac{3}{2}-\frac{\frac{5}{2}}{2x+1}$

$\int\frac{4x^3+2x-1}{2x+1}\,dx = \int 2x^2-x+\frac{3}{2}-\frac{5}{2}\cdot\frac{1}{2x+1}\,dx =$

$= \frac{2}{3}x^3-\frac{x^2}{2}+\frac{3}{2}-\frac{5}{4}\ln|2x+1|+C$