

$$\boxed{11} \text{ a) } \int x \operatorname{sen} x dx = -x \cos x + \int \cos x dx = -x \cos x + \operatorname{sen} x + C$$

$$u = x \rightarrow du = dx \\ dr = \operatorname{sen} x dx \rightarrow r = -\cos x$$

$$\text{b) } \int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C$$

$$u = x \rightarrow du = dx \\ dr = e^x dx \rightarrow r = e^x$$

$$\text{c) } \int (x^2+1) \cos x dx = (x^2+1) \operatorname{sen} x - 2 \int x \operatorname{sen} x dx = (x^2+1) \operatorname{sen} x - 2 \left[-x \cos x + \int \cos x dx \right] =$$

$$u = x^2+1 \rightarrow du = 2x dx \\ dr = \cos x dx \rightarrow r = \operatorname{sen} x$$

$$= (x^2+1) \operatorname{sen} x + 2x \cos x - 2 \operatorname{sen} x + C$$

$$\text{d) } \int \ln x dx = x \ln x - \int \frac{1}{x} \cdot x \cdot dx = x \ln x - \int 1 dx = x \ln x - x + C$$

$$u = \ln x \rightarrow du = \frac{1}{x} dx$$

$$\text{e) } \int \sqrt{x} \ln x dx = \frac{2}{3} \sqrt{x^3} \cdot \ln x - \int \frac{2}{3} x^{\frac{1}{2}} dx = \frac{2}{3} \sqrt{x^3} \ln x - \frac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}+1} =$$

$$u = \ln x \rightarrow du = \frac{1}{x} dx \\ dr = \sqrt{x} dx \rightarrow r = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} = \frac{2}{3} x^{\frac{3}{2}}$$

$$= \frac{2}{3} \sqrt{x^3} \ln x - \frac{4}{9} \sqrt{x^3} + C$$

$$\text{f) } \int \operatorname{arctg} x dx = x \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx = x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) + C$$

$$u = \operatorname{arctg} x \rightarrow du = \frac{dx}{1+x^2}$$

$$dr = dx \rightarrow r = x$$

$$\text{g) } \int x \operatorname{arctg} x dx = \frac{x^2}{2} \operatorname{arctg} x - \int \frac{1}{2} \cdot \frac{x^2+1-1}{1+x^2} dx = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int 1 - \frac{1}{1+x^2} dx =$$

$$u = \operatorname{arctg} x \rightarrow du = \frac{1}{1+x^2} dx \\ dr = x dx \rightarrow r = \frac{x^2}{2}$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} x - \frac{1}{2} \operatorname{arctg} x + C$$

$$\text{h) } \int x^2 \cos(3x) dx = \frac{x^2}{3} \operatorname{sen}(3x) - \frac{2}{3} \int x \operatorname{sen}(3x) dx = \frac{x^2}{3} \operatorname{sen}(3x) - \frac{2}{3} \left[-\frac{1}{3} x \cos(3x) + \frac{1}{3} \int \cos(3x) dx \right] =$$

$$u = x^2 \rightarrow du = 2x dx \\ dr = \cos(3x) dx \rightarrow r = \frac{1}{3} \operatorname{sen}(3x)$$

$$u = x \rightarrow du = dx \\ dr = \operatorname{sen}(3x) dx \rightarrow r = -\frac{1}{3} \cos(3x)$$

$$= \frac{x^2}{3} \operatorname{sen}(3x) + \frac{2}{9} x \cos(3x) - \frac{2}{27} \operatorname{sen}(3x) + C$$

$$\text{i) } \int e^x \operatorname{sen} x dx = -e^x \cos x + \int e^x \cos x dx = -e^x \cos x + e^x \operatorname{sen} x - \int e^x \operatorname{sen} x dx$$

$$u = e^x \rightarrow du = e^x dx \\ dr = \operatorname{sen} x dx \rightarrow r = -\cos x$$

$$u = e^x \rightarrow du = e^x dx \\ dr = \cos x dx \rightarrow r = \operatorname{sen} x$$

Es decir:

$$\int e^x \operatorname{sen} x dx = -e^x \cos x + e^x \operatorname{sen} x - \int e^x \operatorname{sen} x dx \Rightarrow 2 \int e^x \operatorname{sen} x dx = -e^x \cos x + e^x \operatorname{sen} x \Rightarrow$$

$$\Rightarrow \int e^x \operatorname{sen} x dx = \frac{-e^x \cos x + e^x \operatorname{sen} x}{2} + C$$

$$\text{j) } \int \operatorname{sen}^2 x dx = \int \frac{1-\cos(2x)}{2} dx = \frac{1}{2} \int 1 - \frac{1}{2} \cos(2x) dx = \frac{1}{2} \left(x - \frac{1}{2} \operatorname{sen}(2x) \right) = \frac{1}{2} x - \frac{\operatorname{sen}(2x)}{4} + C$$

$$\cos^2 x + \operatorname{sen}^2 x = 1 \\ \cos^2 x - \operatorname{sen}^2 x = \cos(2x) \Rightarrow 2 \operatorname{sen}^2 x = 1 - \cos(2x) \Rightarrow \operatorname{sen}^2 x = \frac{1 - \cos(2x)}{2}$$

También se puede resolver por partes, con $u = \operatorname{sen} x \quad dr = \operatorname{sen} x dx$

$$\therefore \operatorname{sen}(2x) = 2 \operatorname{sen} x \cos x$$

$$\text{k) } \int x^2 e^{-x} dx = -x^2 e^{-x} + \int 2x e^{-x} dx = -x^2 e^{-x} - 2x e^{-x} + \int 2e^{-x} dx = -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C$$

$$u = x^2 \rightarrow du = 2x dx \\ dr = e^{-x} dx \rightarrow r = -e^{-x}$$

$$u = 2x \rightarrow du = 2dx \\ dr = e^{-x} dx \rightarrow r = -e^{-x}$$