SOLUCIÓN REPASO ANÁLISIS

1.- • $\displaystyle\int \frac{e^{3x}}{e^{2x}-3e^x+2}\,dx = \int \frac{e^{2x}}{e^{2x}-3e^x+2}\, e^x dx = \int \frac{t^2}{t^2-3t+2}\,dt = \int\left(1+\frac{3t-2}{t^2-3t+2}\right)dt = \int dt + \int \frac{3t-2}{t^2-3t+2}\,dt =$

$e^x = t \Rightarrow e^x dx = dt$

$t^2 \;\; \underline{| t^2-3t+2}$
$\;\; 1$
$-t^2+3t-2$
$+3t-2$

$\displaystyle \frac{3t-2}{t^2-3t+2} = \frac{A}{t-1} + \frac{B}{t-2} = \frac{A(t-2)+B(t-1)}{(t-1)(t-2)}$

$3t-2 = A(t-2)+B(t-1)$

Si $t=2$ $\;4 = B$

$t=1$ $\;A=-1$

$\displaystyle = \int dt - \int \frac{1}{t-1}\,dt + 4\int \frac{1}{t-2}\,dt =$

$= t - \ln|t-1| + 4\ln|t-2| =$ deshacemos el cambio

$= e^x - \ln|e^x-1| + 4\ln|e^x-2| + C$

• $\displaystyle\int \frac{x}{x^4+16}\,dx = \int \frac{x}{16\left(\frac{x^4}{16}+1\right)}\,dx = \frac{1}{16}\int \frac{1\cdot x}{1+\left(\frac{x^2}{4}\right)^2}\,dx = \frac{1}{16\frac{1}{2}}\int \frac{\frac{1}{2}\cdot x}{1+\left(\frac{x^2}{4}\right)^2}\,dx = \frac{1}{8}\,\text{arctg}\left(\frac{x^2}{4}\right) + C$

• $\displaystyle\int \frac{x^2-2x}{e^x}\,dx = \int (x^2-2x)\cdot e^{-x}\,dx = -(x^2-2x)\,e^{-x} + \int (2x-2)\,e^{-x}\,dx =$

$u = x^2-2x \Rightarrow du = (2x-2)dx$

$dv = e^{-x}dx \Rightarrow v = -e^{-x}$

$u = 2x-2 \Rightarrow du = 2dx$

$dv = e^{-x}dx \Rightarrow v = -e^{-x}$

$\displaystyle = -(x^2-2x)e^{-x} + \left[-(2x-2)\,e^{-x} + \int 2e^{-x}dx\right] = -(x^2-2x)e^{-x} - (2x-2)\,e^{-x} - 2e^{-x}$

$= e^{-x}(-x^2+2x-2x+2-2) = -x^2e^{-x} + C$

• $\displaystyle\int \frac{4x^3+2x-1}{2x+1}\,dx = \int\left(2x^2 - x + \frac{3}{2} + \frac{(-5/2)}{2x+1}\right)dx = 2\int x^2dx - \int x\,dx + \frac{3}{2}dx - \frac{5}{4}\int \frac{2}{2x+1}\,dx$

$4x^3+2x-1 \;\; \underline{| 2x+1}$
$\;\; 2x^2-x+3/2$
$-4x^3-2x^2$
$-2x^2+2x-1$
$+2x^2+x$
$3x-1$
$-3x-3/2$
$-5/2$

$\displaystyle = \frac{2x^3}{3} - \frac{x^2}{2} + \frac{3x}{2} - \frac{5}{4}\ln|2x+1| + C$

2.- $f(x) = x\ln x$

Dom $f(x) = (0,+\infty)$

Ptos de corte: OX $y=0$ $\;x\ln x = 0 \Rightarrow \ln x = 0 \Rightarrow x = 1$ $\;(1,0)$

$\displaystyle\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+}\left(\frac{\ln x}{1/x}\right) \underset{\text{L'H}}{=} \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} \frac{x^2}{-x} = \lim_{x\to 0^+}(-x) = 0$

$\displaystyle\lim_{x\to +\infty} x\ln x = +\infty$

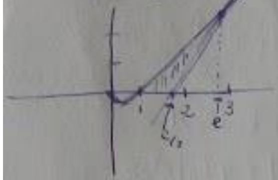
$f'(x) = \ln x + x\cdot\frac{1}{x} = \ln x + 1$ $\quad f'(x) = 0 \Rightarrow \ln x = -1 \Rightarrow x = \frac{1}{e}$

$f''(x) = \frac{1}{x}$ $\quad f''\left(\frac{1}{e}\right) > 0 \Rightarrow x = \frac{1}{e}$ mínimo en $\left(1/e, -1/e\right)$

$f''(x) \neq 0 \Rightarrow$ no tiene ptos de inflexión

Recta tangente de $f(x)$ en $x=e$ $\quad y - f(e) = f'(e)(x-e)$

$$\left.\begin{array}{l} f'(e) = 1+\ln e = 2 \\ f(e) = e \end{array}\right\} \Rightarrow y - e = 2(x-e) \Rightarrow \boxed{y = 2x - e}$$



$$\left.\begin{array}{l} f(x) = x\ln x \\ y = 2x-e \end{array}\right\} x\ln x = 2x - e \iff x = e$$

$\cancel{x(\ln x - 2) = -e}$

Corte de la recta con el eje OX $\quad 2x - e = 0 \Rightarrow x = e/2$

Entonces el área pedida

$$A = \int_1^e x\ln x\,dx - \int_{e/2}^{e}(2x-e)\,dx = \left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_1^e - \left[x^2 - ex\right]_{e/2}^{e} = \left[\frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4}\right] - \left[e^2 - e^2 - \frac{e^2}{4} + \frac{e^2}{2}\right]$$

$u = \ln x \Rightarrow du = \frac{1}{x}dx$

$dv = x\,dx \Rightarrow v = \int x\,dx = \frac{x^2}{2}$

$$= \frac{1}{4}u^2$$

3.- Para que $F(x) = 1 + \operatorname{sen}^2 x$ $\quad G(x) = \frac{-\cos 2x}{2}$ sean primitivas de una misma función, debe cumplir que $F'(x) = G'(x)$

$$\left.\begin{array}{l} F'(x) = 2\operatorname{sen}x\cos x = \operatorname{sen}2x \\ G'(x) = \frac{1}{2}(\operatorname{sen}2x)\cdot 2 = \operatorname{sen}2x \end{array}\right\} \Rightarrow F'(x) = G'(x)$$

por tanto $F(x)$ y $G(x)$ son primitivas de una misma función

4.- $$\int_0^3 \frac{\sqrt{x}}{x+1}\,dx \underset{(*)}{=}$$

cambio $x = t^2 \Rightarrow dx = 2t\,dt$

$$\int \frac{\sqrt{x}\,dx}{x+1} \overset{\substack{\text{cambio} \\ x=t^2}}{=} \int \frac{2t^2\,dt}{t^2+1} = 2\int dt - 2\int \frac{1}{1+t^2} = 2t - \operatorname{arctg}(t) = 2\sqrt{x} - 2\operatorname{arctg}\sqrt{x}$$

Por tanto:

$$(*) = \left[2\sqrt{x} - 2\operatorname{arctg}\sqrt{x}\right]_0^3 = \left(2\sqrt{3} - 2\operatorname{arctg}\sqrt{3}\right) - (0-0) = 2\sqrt{3} - 2\cdot\frac{\pi}{3} = \frac{6\sqrt{3} - 2\pi}{3}$$

GLOBAL - 2ª EVALUACIÓN 28/02/18

- Calcula: $\lim_{x\to 0} \dfrac{2\operatorname{arctg} x - x}{2x - \operatorname{arcsen} x} \overset{0/0\ L'H}{=} \lim_{x\to 0} \dfrac{2\cdot\frac{1}{1+x^2} - 1}{2 - \frac{1}{\sqrt{1-x^2}}} = \dfrac{2\cdot\frac{1}{1+0} - 1}{2 - \frac{1}{\sqrt{1-0}}} = 1$

$\lim_{x\to 0^+} (\ln x \operatorname{tg} x) \underset{-\infty\cdot 0}{=} \lim_{x\to 0^+} \dfrac{\ln x}{\frac{\cos x}{\operatorname{sen} x}} \underset{\infty/\infty}{=} \lim_{x\to 0^+} \dfrac{1/x}{-\frac{1}{\operatorname{sen}^2 x}} = \lim_{x\to 0^+} \dfrac{-\operatorname{sen}^2 x}{x} \underset{0/0}{=} \lim_{x\to 0^+} \dfrac{-2\operatorname{sen} x \cos x}{1} = \dfrac{0}{1} = 0$

- De $f(x)$ se sabe que pasa por el origen de coordenadas y que $f'(x) = \dfrac{1}{1+e^x}$

Encontrar $f(x)$

$f(x)$ es una primitiva de $f'(x)$ además $f(0) = 0$

$$\int \frac{1}{1+e^x}\,dx \underset{\uparrow}{=} \int \frac{1}{1+t}\cdot\frac{dt}{t} \underset{(*)}{=} \int \frac{1}{t}\,dt - \int \frac{1}{t+1} = \ln|t| - \ln|t+1| = (**)$$

$e^x = t \Rightarrow e^x dx = dt \Rightarrow dx = \frac{dt}{t}$

$$\frac{1}{t(1+t)} = \frac{A}{t} + \frac{B}{t+1} = \frac{A(t+1) + B(t)}{t(t+1)}$$

$1 = At + Bt + A$

$0 = A + B$
$1 = A$ $\Big\}$ $B = -1$

$(**) = \ln e^x - \ln(e^x + 1) = x - \ln(e^x + 1) + C$

$f(x) = x \ln(e^x + 1) + C$
Como $f(0) = 0$ $\Big\}$ $0 = 0 - \ln 2 + C$ $C = \ln 2$ $\quad f(x) = x - \ln(1 + e^x) + \ln 2$

2.- Calcula m, n, b para que $f(x) = \begin{cases} mx^2 + nx + 5 & \text{si } x < 1 \\ 3x + 1 & \text{si } x \geq 1 \end{cases}$ cumpla las hipótesis del th de Rolle en el intervalo $[-2, b]$ y determina el valor garantizado por el teorema.

$f(x)$ continua en $[-2, b]$
$f(x)$ derivable en $(-2, b)$ $\Big\} \Rightarrow \exists\, c \in (-2, b) \ /\ f'(c) = 0$
$f(-2) = f(b)$

a) $f(x)$ por ser polinómica, es siempre continua excepto en $x = 1$. Veamos cuando es continua en $x = 1$

$f(1) = 4$

$\lim_{x\to 1} f(x) = \begin{cases} \lim_{x\to 1^+} (3x+1) = 4 \\ \lim_{x\to 1^-} (mx^2 + nx + 5) = m + n + 5 \end{cases} \Big\}\ m + n + 5 = 4 \Rightarrow m + n = -1$

b) $f(x)$ por ser polinómica es derivable salvo en $x = 1$. Para que sea derivable

$f'(1)^+ = \lim_{h\to 0^+} \dfrac{f(1+h) - f(1)}{h} = \lim_{h\to 0^+} \dfrac{3(1+h) + 1 - 4}{h} = \lim_{h\to 0^+} \dfrac{3h}{h} = 3$

$f'(1)^- = \lim_{h\to 0^-} \dfrac{m(1+h)^2 + n(1+h) + 5 - 4}{h} = \lim_{h\to 0^-} \dfrac{mh^2 + 2mh + nh + m + n + 1}{h} = \lim_{h\to 0^-} \dfrac{h(mh + 2m + n)}{h}$

$= \lim_{h\to 0^-} (mh + 2m + n) = 2m + n$

Entonces: $\begin{rcases} m + n = -1 \\ 2m + n = 3 \end{rcases}$ $\boxed{m = 4}$ $\boxed{n = -5}$

c) $f(-2) = f(b)$ $\quad f(-2) = 31$

$$f(b) = \begin{cases} 4b^2-5b+5 & \text{si } -2 < b < 1 \\ 3b+1 & \text{si } b \geq 1 \end{cases}$$

$4b^2-5b+5 = 31 \quad 4b^2-5b-26 = 0 \quad \cancel{b=-2} \quad b = \frac{31}{4}$

$b = \frac{5 \pm \sqrt{25+16\cdot 26}}{2\cdot 4}$

$3b+1 = 31 \Rightarrow b=10$

Entonces $\exists\, c \in (-2,10) \mid f'(c)=0$ $\qquad f'(x) = \begin{cases} 8x-5 & \text{si } x<1 \\ 3 & \text{si } x \geq 1 \end{cases}$

$f'(c)=0 \quad 8c-5=0 \quad \boxed{c = \frac{5}{8} \in (-2,10)}$

3. • Sea f continua y derivable tal que $f(0)=3$. Calcular cuánto tiene que valer $f(5)$ para asegurar ~~que~~ ~~en~~ el intervalo $[0,5]$ existe c tal que $f'(c)=8$. ¿Qué teorema utilizas? Enúncialo y ~~[illegible]~~ indica su interpretación.

Th. del valor medio: f continua y derivable en $[0,5]$ y $(0,5)$, entonces

$\exists\, c \in (0,5) \;/\; f'(c) = \frac{f(5)-f(0)}{5-0} \Rightarrow$

$8 = \frac{f(5)-3}{5} \Rightarrow f(5) = 43$

• Enuncia el th. fundamental del cálculo integral. Calcula la ecuación de ~~[illegible]~~ la recta tangente a la gráfica de $F(x) = \int_0^x (2+\cos t^2)$ en el pto de abscisa $x=0$

$y - F(0) = F'(0)(x-0)$

$F(0) = \int_0^0 (2+\cos t^2)\,dt \quad F'(x) = 2+\cos x^2 \quad F'(0)=3 \quad y = 3x$

4.. Área entre las curvas $f(x) = \frac{-4x}{(1+x^2)^2}$ $\quad g(x) = -x$. Dibuja la situación representando para ello las funciones dadas.

$f(-x) = -f(x)$

$\lim_{x\to\infty} \frac{-4x}{(1+x^2)^2} \overset{L'H}{=} \lim_{x\to\infty} \frac{-4}{2(1+x^2)\cdot 2x} = 0$

$f'(x) = \frac{(1+x^2)^{\cancel{2}}\cdot(-4) + 4x\cdot 2(1+x^2)\cdot 2x}{(1+x^2)^{\cancel{4}\,3}} =$

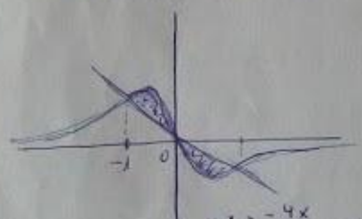
$= \frac{-4-4x^2+16x^2}{(1+x^2)^3} = \frac{12x^2-4}{(1+x^2)^3}$

$f'(x)=0$ si $12x^2-4=0 \quad x^2 = \frac{4}{12} \quad x = \pm\frac{1}{\sqrt{3}}$

$f''(x) = \frac{48x-48x^3}{(1+x^2)^4}$ $\quad f''(1/\sqrt{3})>0 \quad x = 1/\sqrt{3}$ mínimo

$f''(-1/\sqrt{3})<0 \quad x=-1/\sqrt{3}$ máx

$f''(x)=0$ si $x=0$, $x=-1$, $x=1$ $\Big\}$ Pto de inflexión



$-x = \frac{-4x}{(1+x^2)^2} \Rightarrow x(1+x^2)^2 = 4x$

$x\left[(1+x^2)^2-4\right]=0 \rightarrow x=0$; $\rightarrow 1+x^2=2 \Rightarrow x=\pm 1$

$1+x^2=-2$ no posible

$A = \int_0^1 \left(-x - \frac{-4x}{(1+x^2)^2}\right)dx = -\int_0^1 dx + 2\int (1+x^2)^{-2}\,2x\,dx = -\left[\frac{x^2}{2}\right]_0^1 + 2\left[\frac{-1}{1+x^2}\right]_0^1 = 1/2$

$A_T = 1\,u^2$

SOLUCION EXAMEN CALCULO INTEGRAL 13/02/1

1. Se llama <u>integral</u> <u>indefinida</u> de $f(x)$ al conjunto de todas las primitivas de $f(x)$. Se representa $\int f(x) = F(x) + C$ $\quad F(x)$ es una primitiva, C, es una constante

- $$\int x^2 \cos x \, dx = x^2 \operatorname{sen} x - \int 2x \operatorname{sen} x \, dx =$$

  $u = x^2 \Rightarrow 2x\,dx = du$

  $\cos x \, dx = dv \Rightarrow \operatorname{sen} x = v$

  $u = x \to du = dx$

  $\operatorname{sen} x \, dx = dv \to -\cos x = v$

  $$= x^2 \operatorname{sen} x - 2\left[-x \cdot \cos x - \int -\cos x \, dx\right] = x^2 \operatorname{sen} x + 2x\cos x - 2\operatorname{sen} x + C$$

- Regla de Barrow: Si f es continua en $[a,b]$ y $G(x)$ es una primitiva de $f(x)$ en $[a,b]$ $\quad \int_a^b f(x)\,dx = G(b) - G(a)$

- $$\int_1^6 \left(\frac{1}{x} - \ln x\right) dx = \ln|x| - (x\ln x - x)\Big]_1^6 = \left[\ln 6 - 6\ln 6 + 6\right] - \left[\cancel{\ln 1} - (\cancel{1\ln 1} - 1)\right]$$

  $$= \ln 6 - 6\ln 6 + 6 + 1 = -5\ln 6 + 5$$

2. $$\int x \ln(x^2+1)\,dx = \frac{x^2}{2} \cdot \ln(1+x^2) - \int \frac{x^2}{\cancel{2}} \cdot \frac{\cancel{2}x}{1+x^2}\,dx = (*)$$

  $u = \ln(1+x^2) \Rightarrow du = \frac{2x}{1+x^2}dx$

  $x\,dx = dv \Rightarrow \frac{x^2}{2} = v$

  $$\int \frac{x^3}{1+x^2}dx = \int \left(x - \frac{x}{1+x^2}\right)dx = \frac{x^2}{2} - \frac{1}{2}\ln(1+x^2) + C$$

  $$(*) = \frac{x^2}{2}\ln(1+x^2) - \frac{x^2}{2} + \frac{1}{2}\ln(1+x^2) + C$$

- $f(x)$ continua en $[a,b]$ $\quad F(x) = \int_a^x f(t)\,dt \quad F(x)$ es derivable en (a,b) $\boxed{F'(x) = f(x)}$

  $F(x) = \int_0^x f(t)\,dt = x^2(1+x)$ $\quad \left.\begin{array}{l} f(x) = F'(x) \\ F'(x) = 2x + 3x^2 \end{array}\right\}$ $f(2) = F'(2) = 2 \cdot 2 + 3 \cdot 2^2 = \boxed{16}$

3. $$\int_2^3 \frac{5x^3 - 3x + 1}{x^3 - x} = \int_2^3 \left(5 + \frac{2x+1}{x^3 - x}\right)dx = \int_2^3 \left[5 - \frac{1}{x} + \frac{3/2}{x-1} - \frac{1/2}{x+1}\right]dx = (**)$$

  $$\begin{array}{r|l} 5x^3 - 3x + 1 & x^3 - x \\ \cline{2-2} -5x^3 + 5x & 5 \\ \cline{1-1} 2x + 1 & \end{array}$$

  $$\frac{2x+1}{x(x^2-1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

  $2x + 1 = A(x+1)(x-1) + B\,x(x+1) + C(x-1)\cdot x$

  $x=0 \quad 1 = -A$

  $x=1 \quad 3 = 2B$

  $x=-1 \quad -1 = 2C$

  $$(**) = \left[5x - \ln|x| + \frac{3}{2}\ln|x-1| - \frac{1}{2}\ln|x+1|\right]_2^3 =$$

  $$= 15 - \ln 3 + \frac{3}{2}\ln 2 - \frac{1}{2}\ln 4 - \left(10 - \ln 2 - \frac{1}{2}\ln 3\right)$$

  $$= 5 - \tfrac{1}{2}\ln 3 + \tfrac{3}{2}\ln 2$$

$$\int_1^e \frac{(x-1)^2}{x^2+1}\,dx = \int_1^e \frac{x^2+1-2x}{x^2+1}\,dx = \int_1^e \left(\frac{x^2+1}{x^2+1} - \frac{2x}{x^2+1}\right)dx = \int_1^e \left(\frac{x^2+1}{x^2+1}\right)dx - \int \frac{2x}{x^2+1}\,dx$$

$$= \Big[x - \ln(x^2+1)\Big]_1^e = e - L(e^2+1) - (1 - \ln 2) = e - \ln(e^2+1) - 1 + \ln$$

4. $f(x) = x^2 - 2x + 1$

$f'(x) = 2x - 2 \qquad f'(x) = 0 \Leftrightarrow x = 1$

$f''(x) = 2 > 0 \Rightarrow x = 1$ mínimo. $\quad V(1, f(1)) = (1, 0)$

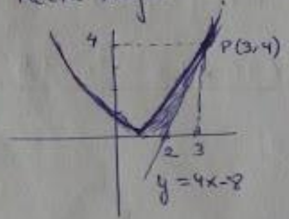
Ptos de corte: Si $x = 0 \quad f(0) = 1 \quad (0, 1)$

Si $y = 0 \quad 0 = x^2 - 2x + 1 \quad x = 1 \quad (1, 0)$

Recta tangente en $(3, 4)$ $\qquad y - 4 = f'(3)(x-3) \qquad f'(3) = 2\cdot 3 - 2 = 4$

$y - 4 = 4(x-3)$

$\boxed{y = 4x - 8}$



$A_{\text{pedida}} = A_{\substack{\text{encerrada} \\ \text{por la curva } y\ x=1\ x=3}} - A_{\text{Triangulo}}$

$= \int_1^3 (x^2 - 2x + 1)\,dx - \frac{1 \times 4}{2}$

$= \left[\frac{x^3}{3} - \frac{2x^2}{2} + x\right]_1^3 - 2 = \frac{2}{3}\,u^2$

$= \frac{27}{3} - 2\cdot\frac{9}{2} + 3 - \left(\frac{1}{3} - 2\cdot\frac{1}{2} + 1\right)$

5. $\int_1^2 f(x)\,dx + \int_2^4 f(x)\,dx = \int_1^4 f(x)\,dx$

$\int_2^4 5 f(x)\,dx = 5\int_2^4 f(x)\,dx$

Entonces: $\quad \int_2^4 5 f(x)\,dx = 5\left[\int_1^4 f(x)\,dx - \int_1^2 f(x)\,dx\right] = 5\,[-4 - 2] = -30$