IES O COUTO. DEPARTAMENTO DE MATEMÁTICAS
Global 1ª Evaluación. Matemáticas I
13-12-2019. Tiempo estimado: 60 minutos

RESOLUCIÓN DEL EXAMEN

Solución del ejercicio 1

$$\frac{8\sqrt{3}}{4-2\sqrt{3}}-\frac{4\sqrt{15}}{\sqrt{5}-\sqrt{3}}-(1+3\sqrt{5})^2=\frac{8\sqrt{3}(4+2\sqrt{3})}{16-12}-\frac{4\sqrt{15}(\sqrt{5}+\sqrt{3})}{5-3}-(1+45+6\sqrt{5})=$$

$$=\frac{8\sqrt{3}(4+2\sqrt{3})}{4}-\frac{4\sqrt{15}(\sqrt{5}+\sqrt{3})}{2}-46-6\sqrt{5}=2\sqrt{3}(4+2\sqrt{3})-2\sqrt{15}(\sqrt{5}+\sqrt{3})-46-6\sqrt{5}=$$

$$8\sqrt{3}+12-10\sqrt{3}-6\sqrt{5}-46-6\sqrt{5}=-2\sqrt{3}-12\sqrt{5}-34$$

Solución del ejercicio 2

a) $z_2^6=(2_{120^\circ})^6=2^6_{6\cdot 120^\circ}=64_{720^\circ}=64_{0^\circ}=64$

b) *Se tiene que* $z_1-\overline{z_1}=2\,Im(z_1)\,i$.

Dado que $Im(z_1)=2\ \text{sen}\ 30^\circ=1$, *entonces:*

$$z_1-\overline{z_1}=2i=2_{90^\circ}\implies(z_1-\overline{z_1})\cdot z_2=2_{90^\circ}\cdot 2_{120^\circ}=4_{210^\circ}=4(\cos 210^\circ+i\ \text{sen}\ 210^\circ)\implies$$

$$\implies(z_1-\overline{z_1})\cdot z_2=4_{210^\circ}=-2\sqrt{3}-2\,i$$

Solución del ejercicio 3

$$(1-i)z^3-2\,i=2\iff(1-i)z^3=2+2\,i\iff z^3=\frac{2+2\,i}{1-i}=\frac{2\sqrt{2}_{45^\circ}}{\sqrt{2}_{-45^\circ}}=2_{90^\circ+360^\circ\,k}$$

Las soluciones son los números complejos $w_k=\sqrt[3]{2}_{30^\circ+120^\circ\,k}$*:*

$$w_0=\sqrt[3]{2}_{30^\circ}=\sqrt[3]{2}\left(\frac{\sqrt{3}}{2}+i\,\frac{1}{2}\right)=\frac{\sqrt[6]{108}}{2}+i\,\frac{\sqrt[3]{2}}{2}$$

$$w_1=\sqrt[3]{2}_{150^\circ}=\sqrt[3]{2}\left(-\frac{\sqrt{3}}{2}+i\,\frac{1}{2}\right)=-\frac{\sqrt[6]{108}}{2}+i\,\frac{\sqrt[3]{2}}{2}$$

$$w_2=\sqrt[3]{2}_{270^\circ}=\sqrt[3]{2}\,i$$

Solución del ejercicio 4

$$\left(\frac{x-1}{x+1}-\frac{x}{x-1}\right):\frac{9x^2-1}{x^4-1}=\left(\frac{(x-1)^2-x(x+1)}{(x+1)(x-1)}\right)\cdot\frac{x^4-1}{9x^2-1}=$$

$$=\left(\frac{x^2-2x+1-x^2-x}{(x+1)(x-1)}\right)\cdot\frac{(x^2+1)(x+1)(x-1)}{(3x-1)(3x+1)}=\frac{(-3x+1)(x^2+1)(x+1)(x-1)}{(x+1)(x-1)(3x-1)(3x+1)}=$$

$$=\frac{(-1)(3x-1)(x^2+1)(x+1)(x-1)}{(x+1)(x-1)(3x-1)(3x+1)}=-\frac{x^2+1}{3x+1}$$

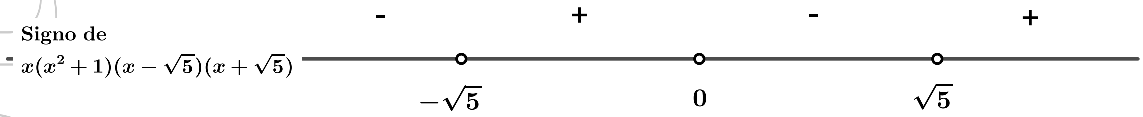
Solución del ejercicio 5

a) $x^5 - 4x^3 - 5x > 0$

1º) $x^5 - 4x^3 - 5x = 0 \iff x(x^4 - 4x^2 - 5) = 0 \iff \begin{cases} x = 0 \\ x^4 - 4x^2 - 5 = 0 \end{cases}$

$x^4 - 4x^2 - 5 = 0 \iff x^2 = \dfrac{4 \pm \sqrt{16+20}}{2} = \begin{cases} x^2 = -1 \\ x^2 = 5 \implies x = \pm\sqrt{5} \end{cases}$

2º) $x^5 - 4x^3 - 5x > 0 \iff x(x^2+1)(x-\sqrt{5})(x+\sqrt{5}) > 0$

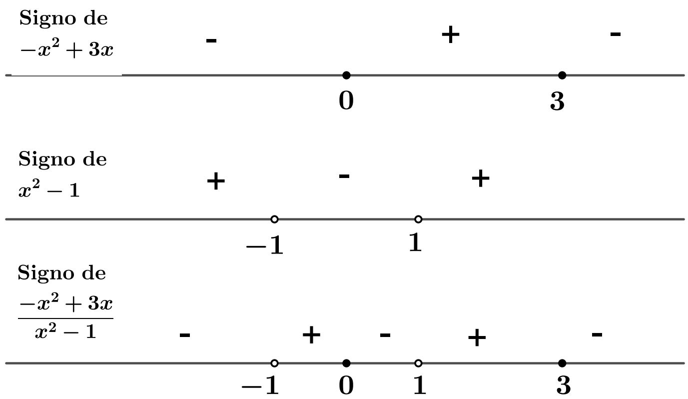


Solución: $(-\sqrt{5}, 0) \cup (\sqrt{5}, +\infty)$

b) $\dfrac{3x-1}{x^2-1} \leq 1 \iff \dfrac{3x-1}{x^2-1} - 1 \leq 0 \iff \dfrac{3x-1-(x^2-1)}{x^2-1} \leq 0 \iff \dfrac{-x^2+3x}{x^2-1} \leq 0$

1º) $-x^2 + 3x = 0 \iff x(-x+3) = 0 \iff \begin{cases} x = 0 \\ x = 3 \end{cases}$

2º) $x^2 - 1 = 0 \iff x^2 = 1 \iff x = \pm 1$



Solución: $(-\infty, -1) \cup [0, 1) \cup [3, +\infty)$

Solución del ejercicio 6

$$\left.\begin{array}{r} x - y + 2z = 6 \\ -x - y + z = 2 \\ 2x + y + z = 3 \end{array}\right\} \xrightarrow[-2F_1+F_3 \to F_3]{F_1+F_2 \to F_2} \left.\begin{array}{r} x - y + 2z = 6 \\ -2y + 3z = 8 \\ 3y - 3z = -9 \end{array}\right\} \xrightarrow{\frac{1}{3}F_3 \to F_3} \left.\begin{array}{r} x - y + 2z = 6 \\ -2y + 3z = 8 \\ y - z = -3 \end{array}\right\} \xrightarrow{2F_3+F_2 \to F_3}$$

$$\left.\begin{array}{r} x - y + 2z = 6 \\ -2y + 3z = 8 \\ z = 2 \end{array}\right\} \implies \begin{cases} z = 2 \\ -2y + 3 \cdot 2 = 8 \implies y = -1 \\ x - (-1) + 2 \cdot 2 = 6 \implies x = 1 \end{cases}$$

Solución: $(1, -1, 2)$

Solución del ejercicio 7

a) $\log(x-1)+\log(2x+2)=0 \implies \log(x-1)(2x+2)=0 \implies$

$$\implies (x-1)(2x+2)=1 \implies 2x^2-2=3 \implies x^2=\frac{3}{2} \implies x=\pm\sqrt{\frac{3}{2}}=\pm\frac{\sqrt{6}}{2}$$

Comprobación de las soluciones:

- $x=\dfrac{\sqrt{6}}{2}$*:*

  $$\log\left(\frac{\sqrt{6}}{2}-1\right)+\log\left(2\frac{\sqrt{6}}{2}+2\right)=\log\left(\frac{\sqrt{6}-2}{2}\right)+\log\left(\sqrt{6}+2\right)=$$

  $$\log\left(\frac{(\sqrt{6}-2)(\sqrt{6}+2)}{2}\right)=\log\left(\frac{6-4}{2}\right)=\log 1=0 \implies x=\frac{\sqrt{6}}{2} \text{ válida.}$$

- $x=-\dfrac{\sqrt{6}}{2}$*:*

  $$\nexists \log\left(-\frac{\sqrt{6}}{2}-1\right) \implies x=-\frac{\sqrt{6}}{2} \text{ no válida.}$$

b) $\begin{cases}\sqrt{y-1}+x=5\\ 2x-y=1\end{cases}$

1º) $2x-y=1 \implies y=2x-1$

2º) $\sqrt{y-1}+x=5 \overset{y=2x-1}{\implies} \sqrt{2x-1-1}+x=5 \implies \sqrt{2x-2}=5-x \implies$

$$\implies 2x-2=(5-x)^2 \implies 2x-2=x^2-10x+25 \implies 0=x^2-12x+27 \implies$$

$$\implies x=\frac{12\pm\sqrt{144-108}}{2}=\begin{cases}x=3\\ x=9\end{cases}$$

3º) $\begin{cases}x=3 \implies y=2\cdot 3-1=5\\ x=9 \implies y=2\cdot 9-1=17\end{cases}$

Comprobación de las soluciones (el problema estaría en la primera ecuación del sistema, que es la que tiene radicales):

- $(3,5)$*:*

  $\sqrt{5-1}+3=2+3=5 \implies (3,5)$ *es válida.*

- $(9,17)$*:*

  $\sqrt{17-1}+9=4+9=13\neq 5 \implies (9,17)$ *no es válida.*