Examen global 1ª Evaluación Matemáticas I (13-12-2019)

GLOBAL 1º EVALUACIÓN 13-12-2019

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$$\frac{8\sqrt{3}}{4-2\sqrt{3}} - \frac{4\sqrt{15}}{\sqrt{5}-\sqrt{3}} - \left(1+3\sqrt{5}\right)^2 =$$

$$= \frac{8\sqrt{3}\left(4+2\sqrt{3}\right)}{4^2-\left(2\sqrt{3}\right)^2} - \frac{4\sqrt{15}\left(\sqrt{5}+\sqrt{3}\right)}{5-3} - \left(1+9\cdot 5+6\sqrt{5}\right) =$$

$$= \frac{8\sqrt{3}\left(4+2\sqrt{3}\right)}{16-12} - \frac{4\sqrt{15}\left(\sqrt{5}+\sqrt{3}\right)}{2} - \left(46+6\sqrt{5}\right) =$$

$$= \frac{8\sqrt{3}\left(4+2\sqrt{3}\right)}{4} - 2\sqrt{15}\left(\sqrt{5}+\sqrt{3}\right) - 46 - 6\sqrt{5} =$$

$$= 2\sqrt{3}\left(4+2\sqrt{3}\right) - 2\cdot 5\sqrt{3} - 2\cdot 3\cdot\sqrt{5} - 46 - 6\sqrt{5} =$$

$$= 8\sqrt{3} + 12 - 10\sqrt{3} - 6\sqrt{5} - 46 - 6\sqrt{5} =$$

$$= \boxed{-2\sqrt{3} - 12\sqrt{5} - 34}$$

2 a) $z_2^6 = \left(2_{120^\circ}\right)^6 = 2^6_{720^\circ} = \boxed{64_{360^\circ} = 64}$

$$\left.\begin{array}{l} |z_2| = \sqrt{(-1)^2+(\sqrt{3})^2} = 2 \\ \frac{Im(z_2)}{Re(z_2)} = -\sqrt{3} \end{array}\right\} \Longrightarrow z_2 = 2_{120^\circ}$$

b) $(z_1 - \bar{z}_1)\, z_2 = 2_{90^\circ} \cdot 2_{120^\circ} = 4_{210^\circ}$

$z_1 - \bar{z}_1 = 2i\, Im(z_1) = i\,4\,sen 30^\circ = 2i = 2_{90^\circ}$

$z_1 = 2\cos 30^\circ + i\,2\,sen 30^\circ$

$z_2 = 2_{120^\circ}$

$$\boxed{(z_1 - \bar{z}_1)\, z_2 = 4_{210^\circ}}$$

$$(z_1 - \bar{z}_1)\, z_2 = 4\cos 210^\circ + i\,4\,sen 210^\circ = 4\cdot\left(-\frac{\sqrt{3}}{2}\right) + i\cdot 4\cdot\left(-\frac{1}{2}\right)$$

$$= -2\sqrt{3}\,i - 2i$$

$\boxed{3}$ $(1-i)z^3 - 2i = 2 \iff (1-i)z^3 = 2+2i \iff$

$$\iff z^3 = \frac{2+2i}{1-i} = \frac{2\sqrt{2}_{45^\circ}}{\sqrt{2}_{-45^\circ}}$$

$$2+2i = 2\sqrt{2}_{45^\circ}$$
$$|2+2i| = 2\sqrt{2}$$
$$\frac{Im(2+2i)}{Re(2+2i)} = 1$$

$$1-i = \sqrt{2}_{-45^\circ}$$
$$|1-i| = \sqrt{2}$$
$$\frac{Im(1-i)}{Re(1-i)} = -1$$

$$z^3 = \frac{2\sqrt{2}_{45^\circ}}{\sqrt{2}_{-45^\circ}} = 2_{90^\circ+360^\circ k} \Longrightarrow$$

$$\Longrightarrow \quad w_k = \sqrt[3]{2}_{30^\circ+120^\circ k}$$

$$w_0 = \sqrt[3]{2}_{30^\circ} = \sqrt[3]{2}(\cos 30^\circ + i \operatorname{sen} 30^\circ)$$
$$w_1 = \sqrt[3]{2}_{150^\circ} = \sqrt[3]{2}(\cos 150^\circ + i \operatorname{sen} 150^\circ)$$
$$w_2 = \sqrt[3]{2}_{270^\circ} = -\sqrt[3]{2}\, i$$

$$\sqrt[3]{2}\cdot\sqrt{3} = \sqrt[6]{2^2\cdot 3^3} = \sqrt[6]{108}$$

$$\Longrightarrow \boxed{w_0 = \frac{\sqrt[3]{2}}{2}\cdot\sqrt{3} + \frac{\sqrt[3]{2}}{2}i = \frac{\sqrt[6]{108}}{2} + \frac{\sqrt[3]{2}}{2}i}$$

$$\boxed{w_1 = -\frac{\sqrt[6]{108}}{2} + \frac{\sqrt[3]{2}}{2}i}$$

$$\boxed{w_2 = -\sqrt[3]{2}\, i}$$

$\boxed{4}$

$$\left(\frac{x-1}{x+1}-\frac{x}{x-1}\right):\frac{9x^2-1}{x^4-1}=$$

$$=\left(\frac{(x-1)^2-x(x+1)}{(x+1)(x-1)}\right)\cdot\frac{(x^4-1)}{(9x^2-1)}=$$

$$=\frac{(x^2-2x+1-x^2-x)}{(x+1)(x-1)}\cdot\frac{(x^2+1)(x-1)(x+1)}{(3x-1)(3x+1)}=$$

$$=\frac{(-3x+1)(x^2+1)\cancel{(x-1)}\cancel{(x+1)}}{\cancel{(x-1)}\cancel{(x+1)}(3x-1)(3x+1)}=\frac{(-1)\cancel{(3x-1)}(x^2+1)}{(3x-1)\cancel{(3x+1)}}=$$

$$=\boxed{-\frac{x^2+1}{3x-1}}$$

$\boxed{5}$ a) $x^5-4x^3-5x>0$

1º) $x(x^4-4x^2-5)=0$

$x=0$,

$$x^4-4x^2-5=0 \Longleftrightarrow x^2=\frac{4\pm\sqrt{16+20}}{2}$$

$$\Longleftrightarrow x^2=\frac{4\pm 6}{2}=\begin{cases}5\\-1\end{cases}\qquad \begin{matrix}x^2=5\Rightarrow x=\pm\sqrt{5}\\ x^2=-1\end{matrix}$$

$$x^5-4x^3-5x=x(x-\sqrt{5})(x+\sqrt{5})(x^2+1)$$

SIGNO		$-\sqrt{5}$		0		$\sqrt{5}$	
x^5-4x^3-5x	$-$		$+$		$-$		$+$

Solución: $\boxed{(-\sqrt{5}, 0) \quad \cup \quad (\sqrt{5}, +\infty)}$

b) $\dfrac{3x-1}{x^2-1} \leq 1 \Longleftrightarrow \dfrac{3x-1}{x^2-1} - 1 \leq 0$

$\Longleftrightarrow \dfrac{3x-1-(x^2-1)}{x^2-1} \leq 0 \Longleftrightarrow$

$\Longleftrightarrow \boxed{\dfrac{-x^2+3x}{x^2-1} \leq 0}$

1º) $-x^2+3x=0 \Longleftrightarrow x(-x+3)=0 \Longleftrightarrow$

$\Longleftrightarrow \begin{cases} x=0 \\ x=3 \end{cases}$

2º) $x^2-1=0 \Longleftrightarrow x^2=1 \Longleftrightarrow x=\pm 1$

SIGNO $-x^2+3x$

$-$	0	$+$	3	$-$

SIGNO x^2-1

$+$	-1	$-$	1	$+$

SIGNO $\dfrac{-x^2+3x}{x^2-1}$

$-$	-1	$+$	0	$-$	1	$+$	3	$-$

Solución $\boxed{(-\infty, -1) \cup [0, 1) \cup [3, +\infty)}$

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$$\left\{\begin{array}{l} x - y + 2z = 6 \\ -x - y + z = 2 \\ 2x + y + z = 3 \end{array}\right. \xrightarrow[-2F_1 + F_3 \to F_3]{F_1 + F_2 \to F_2} \left\{\begin{array}{l} x - y + 2z = 6 \\ -2y + 3z = 8 \\ 3y - 3z = -9 \end{array}\right.$$

$$\xrightarrow{\frac{1}{3}F_3 \to F_3} \left\{\begin{array}{l} x - y + 2z = 6 \\ -2y + 3z = 8 \\ y - z = -3 \end{array}\right. \xrightarrow{2F_3 + F_2 \to F_3}$$

$$\left\{\begin{array}{l} x - y + 2z = 6 \\ -2y + 3z \quad 8 \\ z = 2 \end{array}\right. \Longrightarrow$$

1º) $z = 2$

2º) $-2y + 3 \cdot 2 = 8 \Rightarrow \Rightarrow \cancel{y=1}$ $y = -1$

3º) $x + 1 + 2 \cdot 2 = 6 \Rightarrow \Rightarrow x = 1$

Solución: $(1, -1, 2)$

7 a) $\log(x-1) + \log(2x+2) = 0 \Rightarrow$

$\Rightarrow \log\left((x-1)(2x+2)\right) = 0 \Rightarrow$

$\Rightarrow (x-1)(2x+2) = 1 \Rightarrow$

$2x^2 - 2 = 1 \Rightarrow x^2 = \frac{3}{2} \Rightarrow x = \pm\sqrt{\frac{3}{2}}$

$\Rightarrow x = \pm\frac{\sqrt{6}}{2}$

Comprobación:

- $x = \frac{\sqrt{6}}{2}$ (Válida):

$\log\left(\frac{\sqrt{6}}{2} - 1\right) + \log\left(2 \cdot \frac{\sqrt{6}}{2} + 2\right) =$

$= \log\left(\frac{\sqrt{6}-2}{2}\right) + \log\left(\frac{2\sqrt{6}+4}{2}\right) = \log\left(\frac{(\sqrt{6}-2)(2\sqrt{6}+4)}{4}\right) =$

$$= \log\left(\frac{2\cdot 6 + 4\sqrt{6} - 4\sqrt{6} - 8}{4}\right) = \log 1 = 0$$ ✓

- $x = -\frac{\sqrt{6}}{2}$ (No válida)

$\nexists \log\left(-\frac{\sqrt{6}}{2} - 1\right)$

b) $\begin{cases} \sqrt{y-1} + x = 5 \\ 2x - y = 1 \end{cases}$

1º) $2x - y = 1 \Longrightarrow y = 2x - 1$

2º) $\sqrt{y-1} + x = 5 \xrightarrow{y = 2x-1} \sqrt{2x-1-1} + x = 5$

$\Longrightarrow \sqrt{2x-2} + x = 5 \Longrightarrow \sqrt{2x-2} = 5 - x$

$\sqrt{2x-2} = 5 - x$

$\left(\sqrt{2x-2}\right)^2 = (5-x)^2$

$2x - 2 = 25 - 10x + x^2$

$\cancel{0 = 25}$

$0 = 27 - 12x + x^2$

$x = \frac{12 \pm \sqrt{144 - 108}}{2} = \frac{12 \pm \sqrt{36}}{2} = \frac{12 \pm 6}{2}$

$\Longrightarrow x = \begin{cases} 9 \\ 3 \end{cases}$

3º) $y = 2x - 1$

$x = 9 \Longrightarrow y = 17$

$x = 3 \Longrightarrow 5$

Comprobación (hay radicales en el ejercicio):

- $(9, 17)$: (No válida)

  $\sqrt{17 - 1} + 3 = 4 + 3 \neq 5$

- $(3, 5)$: (Válida)

  $\sqrt{5 - 1} + 3 = 2 + 3 = 5$ ✓