

Examen global 1ª Evaluación Matemáticas I (13-12-2019)

GLOBAL 1ª EVALUACIÓN = 13-12-2019

1)

$$\frac{8\sqrt{3}}{4-2\sqrt{3}} - \frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}} - (1+3\sqrt{5})^2 =$$

$$= \frac{8\sqrt{3}(4+2\sqrt{3})}{4^2 - (2\sqrt{3})^2} - \frac{4\sqrt{15}(\sqrt{5}+\sqrt{3})}{5-3} - (1+9\cdot 5 + 6\sqrt{5}) =$$

$$= \frac{8\sqrt{3}(4+2\sqrt{3})}{16-12} - \frac{4\sqrt{15}(\sqrt{5}+\sqrt{3})}{2} - (46+6\sqrt{5}) =$$

$$= \frac{8\sqrt{3}(4+2\sqrt{3})}{4} - 2\sqrt{15}(\sqrt{5}+\sqrt{3}) - 46 - 6\sqrt{5} =$$

$$= 2\sqrt{3}(4+2\sqrt{3}) - 2\cdot 3\cdot \sqrt{5} - 46 - 6\sqrt{5} =$$

$$= 8\sqrt{3} + 12 - 10\sqrt{3} - 6\sqrt{5} - 46 - 6\sqrt{5} =$$

$$= \boxed{-2\sqrt{3} - 12\sqrt{5} - 34}$$

2) a)  $z_2^6 = (2_{120^\circ})^6 = 2_{720^\circ}^6 = \boxed{64_{360^\circ} = 64}$

$$|z_2| = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$$

$$\frac{\text{Im}(z_2)}{\text{Re}(z_2)} = -\sqrt{3} \Rightarrow z_2 = 2_{120^\circ}$$

b)  $(z_1 - \bar{z}_1)z_2 = 2_{90^\circ} \cdot 2_{120^\circ} = 4_{210^\circ}$

$$z_1 - \bar{z}_1 = 2i \text{Im}(z_1) = i 4 \text{sen } 30^\circ = 2i = 2_{90^\circ}$$

$$z_1 = 2 \cos 30^\circ + i 2 \text{sen } 30^\circ$$

$$z_2 = 2_{120^\circ}$$

$$\boxed{(z_1 - \bar{z}_1)z_2 = 4_{210^\circ}}$$

$$\boxed{(z_1 - \bar{z}_1)z_2 = 4 \cos 210^\circ + i 4 \text{sen } 210^\circ = 4 \cdot \left(-\frac{\sqrt{3}}{2}\right) + i \cdot 4 \cdot \left(-\frac{1}{2}\right)}$$

$$= \boxed{-2\sqrt{3}i - 2i}$$

$$\boxed{3} \quad (1-i)z^3 - 2i = 2 \iff (1-i)z^3 = 2+2i \iff$$

$$\iff z^3 = \frac{2+2i}{1-i} = \frac{2\sqrt{2} \angle 45^\circ}{\sqrt{2} \angle -45^\circ}$$

$$2+2i = 2\sqrt{2} \angle 45^\circ$$

$$|2+2i| = 2\sqrt{2}$$

$$\frac{\text{Im}(2+2i)}{\text{Re}(2+2i)} = 1$$

$$\frac{\text{Im}(1-i)}{\text{Re}(1-i)} = -1$$

$$1-i = \sqrt{2} \angle -45^\circ$$

$$|1-i| = \sqrt{2}$$

$$\frac{\text{Im}(1-i)}{\text{Re}(1-i)} = -1$$

$$\frac{\text{Im}(1-i)}{\text{Re}(1-i)} = -1$$

$$z^3 = \frac{2\sqrt{2} \angle 45^\circ}{\sqrt{2} \angle -45^\circ} = 2 \angle 90^\circ + 360^\circ k$$

$$\implies w_k = \sqrt[3]{2} \angle 30^\circ + 120^\circ k$$

$$w_0 = \sqrt[3]{2} \angle 30^\circ = \sqrt[3]{2} (\cos 30^\circ + i \sin 30^\circ)$$

$$w_1 = \sqrt[3]{2} \angle 150^\circ = \sqrt[3]{2} (\cos 150^\circ + i \sin 150^\circ)$$

$$w_2 = \sqrt[3]{2} \angle 270^\circ = -\sqrt[3]{2} i$$

$$\sqrt[3]{2} \cdot \sqrt{3} = \sqrt[6]{2^2 \cdot 3^3} = \sqrt[6]{108}$$

$$\implies w_0 = \frac{\sqrt[3]{2}}{2} \cdot \sqrt{3} + \frac{\sqrt[3]{2}}{2} i = \frac{\sqrt[6]{108}}{2} + \frac{\sqrt[3]{2}}{2} i$$

$$w_1 = -\frac{\sqrt[6]{108}}{2} + \frac{\sqrt[3]{2}}{2} i$$

$$w_2 = -\sqrt[3]{2} i$$

4

$$\left( \frac{x-1}{x+1} - \frac{x}{x-1} \right) : \frac{9x^2-1}{x^4-1} =$$

$$= \left( \frac{(x-1)^2 - x(x+1)}{(x+1)(x-1)} \right) \cdot \frac{(x^4-1)}{(9x^2-1)} =$$

$$= \frac{(x^2-2x+1 - x^2-x)}{(x+1)(x-1)} \cdot \frac{(x^2+1)(x-1)(x+1)}{(3x-1)(3x+1)} =$$

$$= \frac{(-3x+1)(x^2+1)(x-1)(x+1)}{(x-1)(x+1)(3x-1)(3x+1)} = \frac{(-1)(3x-1)(x^2+1)}{(3x-1)(3x+1)} =$$

$$= \left[ -\frac{x^2+1}{3x+1} \right]$$

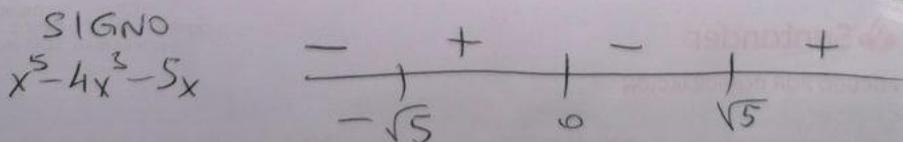
5 a)  $x^5 - 4x^3 - 5x > 0$

1°)  $x(x^4 - 4x^2 - 5) = 0$

$x^4 - 4x^2 - 5 = 0 \iff x^2 = \frac{4 \pm \sqrt{16+20}}{2}$

$\iff x^2 = \frac{4 \pm 6}{2} = \begin{cases} 5 & x^2=5 \Rightarrow x=\pm\sqrt{5} \\ -1 & x^2=-1 \end{cases}$

$x^5 - 4x^3 - 5x = x(x-\sqrt{5})(x+\sqrt{5})(x^2+1)$



Solución:  $(-\sqrt{5}, 0) \cup (\sqrt{5}, +\infty)$

$$b) \frac{3x-1}{x^2-1} \leq 1 \iff \frac{3x-1}{x^2-1} - 1 \leq 0$$

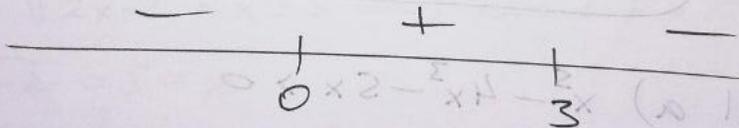
$$\iff \frac{3x-1 - (x^2-1)}{x^2-1} \leq 0$$

$$\iff \frac{-x^2 + 3x}{x^2-1} \leq 0$$

$$1) -x^2 + 3x = 0 \iff x(-x+3) = 0 \iff \begin{cases} x_1 = 0 \\ x_2 = 3 \end{cases}$$

$$2) x^2 - 1 = 0 \iff x^2 = 1 \iff x = \pm 1$$

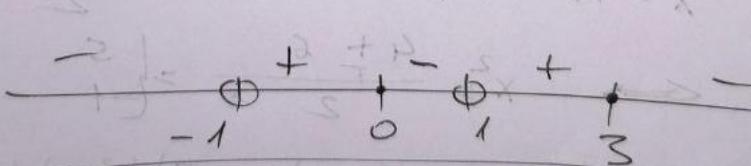
SIGNO  
 $-x^2 + 3x$



SIGNO  
 $x^2 - 1$



SIGNO  
 $\frac{-x^2 + 3x}{x^2 - 1}$



Solución:  $(-\infty, -1) \cup [0, 1) \cup [3, +\infty)$

6

$$\begin{cases} x - y + 2z = 6 & F_1 + F_2 \rightarrow F_2 \\ -x - y + z = 2 & \\ 2x + y + z = 3 & -2F_1 + F_3 \rightarrow F_3 \end{cases} \begin{cases} x - y + 2z = 6 \\ -2y + 3z = 8 \\ 3y - 3z = -9 \end{cases}$$
$$\xrightarrow{\frac{1}{3}F_3 \rightarrow F_3} \begin{cases} x - y + 2z = 6 \\ -2y + 3z = 8 \\ y - z = -3 \end{cases} \xrightarrow{2F_3 + F_2 \rightarrow F_2} \begin{cases} x - y + 2z = 6 \\ -2y + 3z = 8 \\ y - z = -3 \end{cases}$$

$$\begin{cases} x - y + 2z = 6 \\ -2y + 3z = 8 \\ z = 2 \end{cases} \begin{array}{l} 1^\circ) \quad z = 2 \\ 2^\circ) \quad -2y + 3 \cdot 2 = 8 \Rightarrow \\ \quad \quad \quad \Rightarrow \quad y = -1 \\ 3^\circ) \quad x + 1 + 2 \cdot 2 = 6 \Rightarrow \\ \quad \quad \quad \Rightarrow \quad x = 1 \end{array}$$

Solución:  $(1, -1, 2)$

$$\begin{aligned} \text{7) a) } \log(x-1) + \log(2x+2) &= 0 \Rightarrow \\ \Rightarrow \log(x-1)(2x+2) &= 0 \Rightarrow \\ \Rightarrow (x-1)(2x+2) &= 1 \Rightarrow \\ 2x^2 - 2 &= 1 \Rightarrow x^2 = \frac{3}{2} \Rightarrow x = \pm \sqrt{\frac{3}{2}} \\ \Rightarrow x &= \pm \frac{\sqrt{6}}{2} \end{aligned}$$

Comprobación:

$x = \frac{\sqrt{6}}{2}$  (Válida):

$$\begin{aligned} \log\left(\frac{\sqrt{6}}{2} - 1\right) + \log\left(2 \cdot \frac{\sqrt{6}}{2} + 2\right) &= \\ = \log\left(\frac{\sqrt{6}-2}{2}\right) + \log\left(\frac{2\sqrt{6}+4}{2}\right) &= \log\left(\frac{(\sqrt{6}-2)(2\sqrt{6}+4)}{4}\right) = \end{aligned}$$

$$= \log \left( \frac{2 \cdot 6 + 4\sqrt{6} - 4\sqrt{6} - 8}{4} \right) = \log 1 = 0 \quad \checkmark$$

•  $x = -\frac{\sqrt{6}}{2}$  (No válida)

~~$\exists \log \left( -\frac{\sqrt{6}}{2} - 1 \right)$~~

b) 
$$\begin{cases} \sqrt{y-1} + x = 5 \\ 2x - y = 1 \end{cases}$$

1º)  $2x - y = 1 \implies y = 2x - 1$

2º)  $\sqrt{y-1} + x = 5 \xrightarrow{y=2x-1} \sqrt{2x-1-1} + x = 5$

$\implies \sqrt{2x-2} + x = 5 \implies \sqrt{2x-2} = 5 - x$

$\sqrt{2x-2} = 5 - x$

$(\sqrt{2x-2})^2 = (5-x)^2$

$2x-2 = 25 - 10x + x^2$

~~$0 = 25$~~

$0 = 27 - 12x + x^2$

$x = \frac{12 \pm \sqrt{144 - 108}}{2} = \frac{12 \pm \sqrt{36}}{2} = \frac{12 \pm 6}{2}$

$\implies x = \begin{cases} 9 \\ 3 \end{cases}$

$(4+5x)(3-x) = (4+5x)gal + (3-x)gal =$

$$3^{\circ}) \quad y = 2x - 1$$

$$x = 9 \implies y = 17$$

$$x = 3 \implies 5$$

Comprobación (hay radicales en el ejercicio):

- $(9, 17)$ : (No válida)

$$\sqrt{17-1} + 3 = 4 + 3 \neq 5$$

- $(3, 5)$ : (Válida)

$$\sqrt{5-1} + 3 = 2 + 3 = 5 \quad \checkmark$$