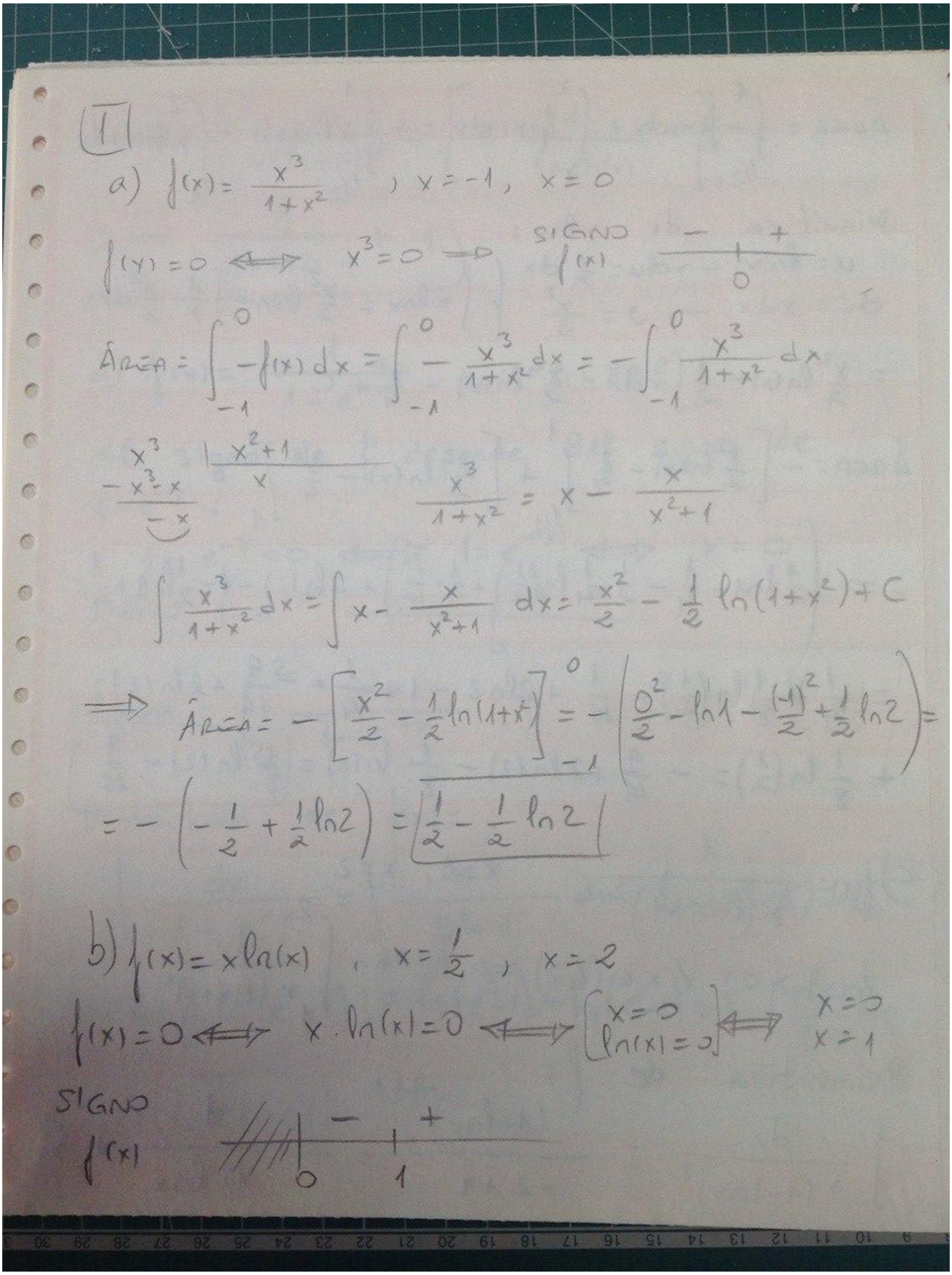
SOLUCIONES A LOS EJERCICIOS DEL BOLETÍN DE REPASO DE CÁLCULO INTEGRAL (2019-2020)



$\boxed{1}$

a) $f(x) = \dfrac{x^3}{1+x^2}$, $x=-1$, $x=0$

$f(x)=0 \Longleftrightarrow x^3=0 \Rightarrow$ SIGNO $f(x)$ $\begin{array}{ccc} - & & + \\ \hline & 0 & \end{array}$

$$\text{ÁREA} = \int_{-1}^{0} -f(x)\,dx = \int_{-1}^{0} -\frac{x^3}{1+x^2}\,dx = -\int_{-1}^{0} \frac{x^3}{1+x^2}\,dx$$

$$\begin{array}{r|l} x^3 & x^2+1 \\ \cline{2-2} -x^3-x & x \\ \hline -x & \end{array} \qquad \frac{x^3}{1+x^2} = x - \frac{x}{x^2+1}$$

$$\int \frac{x^3}{1+x^2}\,dx = \int x - \frac{x}{x^2+1}\,dx = \frac{x^2}{2} - \frac{1}{2}\ln(1+x^2) + C$$

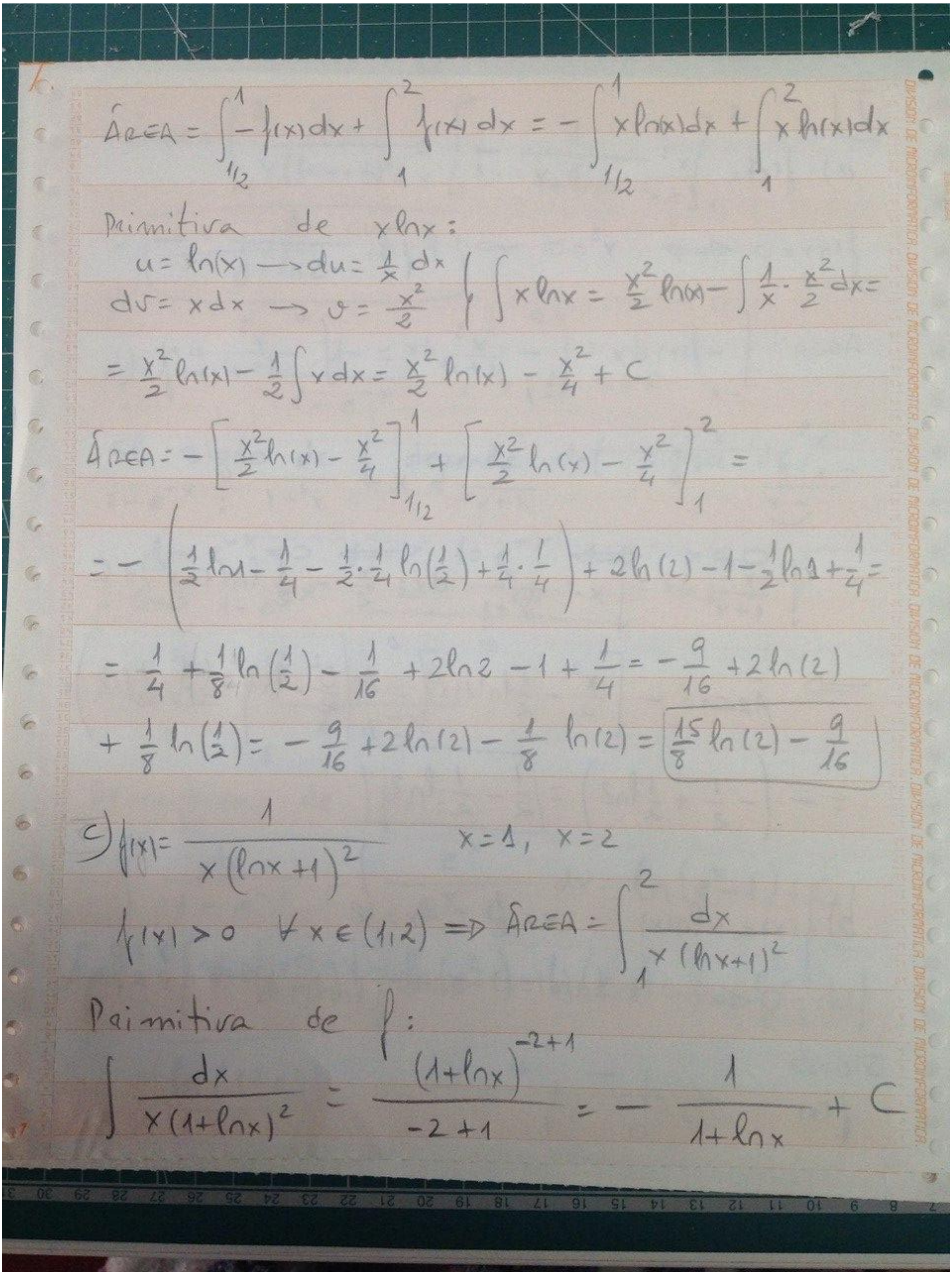
$$\Longrightarrow \text{ÁREA} = -\left[\frac{x^2}{2} - \frac{1}{2}\ln(1+x^2)\right]_{-1}^{0} = -\left(\frac{0^2}{2} - \ln 1 - \frac{(-1)^2}{2} + \frac{1}{2}\ln 2\right) =$$

$$= -\left(-\frac{1}{2} + \frac{1}{2}\ln 2\right) = \boxed{\frac{1}{2} - \frac{1}{2}\ln 2}$$

b) $f(x) = x\ln(x)$, $x = \dfrac{1}{2}$, $x=2$

$$f(x)=0 \Longleftrightarrow x\cdot\ln(x)=0 \Longleftrightarrow \left[\begin{array}{l} x=0 \\ \ln(x)=0 \end{array}\right] \Longleftrightarrow \begin{array}{l} x=0 \\ x=1 \end{array}$$

SIGNO $f(x)$ $\begin{array}{cccc} ////// & - & & + \\ \hline 0 & & 1 & \end{array}$



$$\text{ÁREA} = \int_{1/2}^{1} -f(x)\,dx + \int_{1}^{2} f(x)\,dx = -\int_{1/2}^{1} x\ln(x)\,dx + \int_{1}^{2} x\ln(x)\,dx$$

Primitiva de $x\ln x$:

$u = \ln(x) \longrightarrow du = \frac{1}{x}dx$

$dv = x\,dx \longrightarrow v = \frac{x^2}{2}$

$$\int x\ln x = \frac{x^2}{2}\ln(x) - \int \frac{1}{x}\cdot\frac{x^2}{2}dx =$$

$$= \frac{x^2}{2}\ln(x) - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln(x) - \frac{x^2}{4} + C$$

$$\text{ÁREA} = -\left[\frac{x^2}{2}\ln(x) - \frac{x^2}{4}\right]_{1/2}^{1} + \left[\frac{x^2}{2}\ln(x) - \frac{x^2}{4}\right]_{1}^{2} =$$

$$= -\left(\frac{1}{2}\ln 1 - \frac{1}{4} - \frac{1}{2}\cdot\frac{1}{4}\ln\left(\frac{1}{2}\right) + \frac{1}{4}\cdot\frac{1}{4}\right) + 2\ln(2) - 1 - \frac{1}{2}\ln 1 + \frac{1}{4} =$$

$$= \frac{1}{4} + \frac{1}{8}\ln\left(\frac{1}{2}\right) - \frac{1}{16} + 2\ln 2 - 1 + \frac{1}{4} = -\frac{9}{16} + 2\ln(2)$$

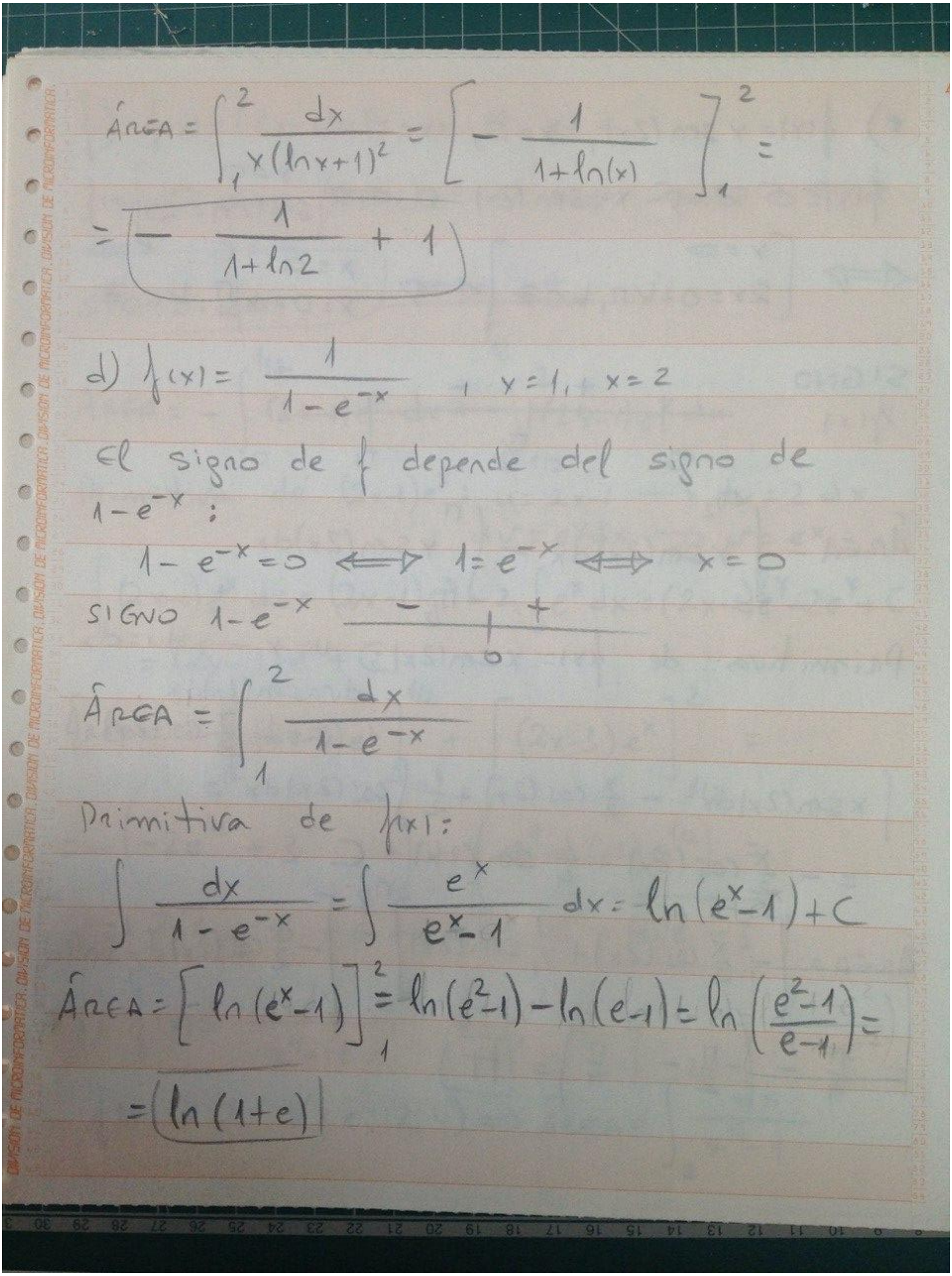
$$+ \frac{1}{8}\ln\left(\frac{1}{2}\right) = -\frac{9}{16} + 2\ln(2) - \frac{1}{8}\ln(2) = \boxed{\frac{15}{8}\ln(2) - \frac{9}{16}}$$

c) $f(x) = \dfrac{1}{x(\ln x + 1)^2}$ $\quad x = 1,\ x = 2$

$$f(x) > 0 \quad \forall x \in (1,2) \Rightarrow \text{ÁREA} = \int_{1}^{2} \frac{dx}{x(\ln x + 1)^2}$$

Primitiva de f:

$$\int \frac{dx}{x(1+\ln x)^2} = \frac{(1+\ln x)^{-2+1}}{-2+1} = -\frac{1}{1+\ln x} + C$$



ÁREA $= \int_1^2 \frac{dx}{x(\ln x+1)^2} = \left[-\frac{1}{1+\ln(x)}\right]_1^2 =$

$= \boxed{-\frac{1}{1+\ln 2}+1}$

d) $f(x)=\frac{1}{1-e^{-x}}$, $x=1$, $x=2$

El signo de f depende del signo de $1-e^{-x}$:

$1-e^{-x}=0 \Longleftrightarrow 1=e^{-x} \Longleftrightarrow x=0$

SIGNO $1-e^{-x}$ $\quad - \quad \underset{0}{|} \quad +$

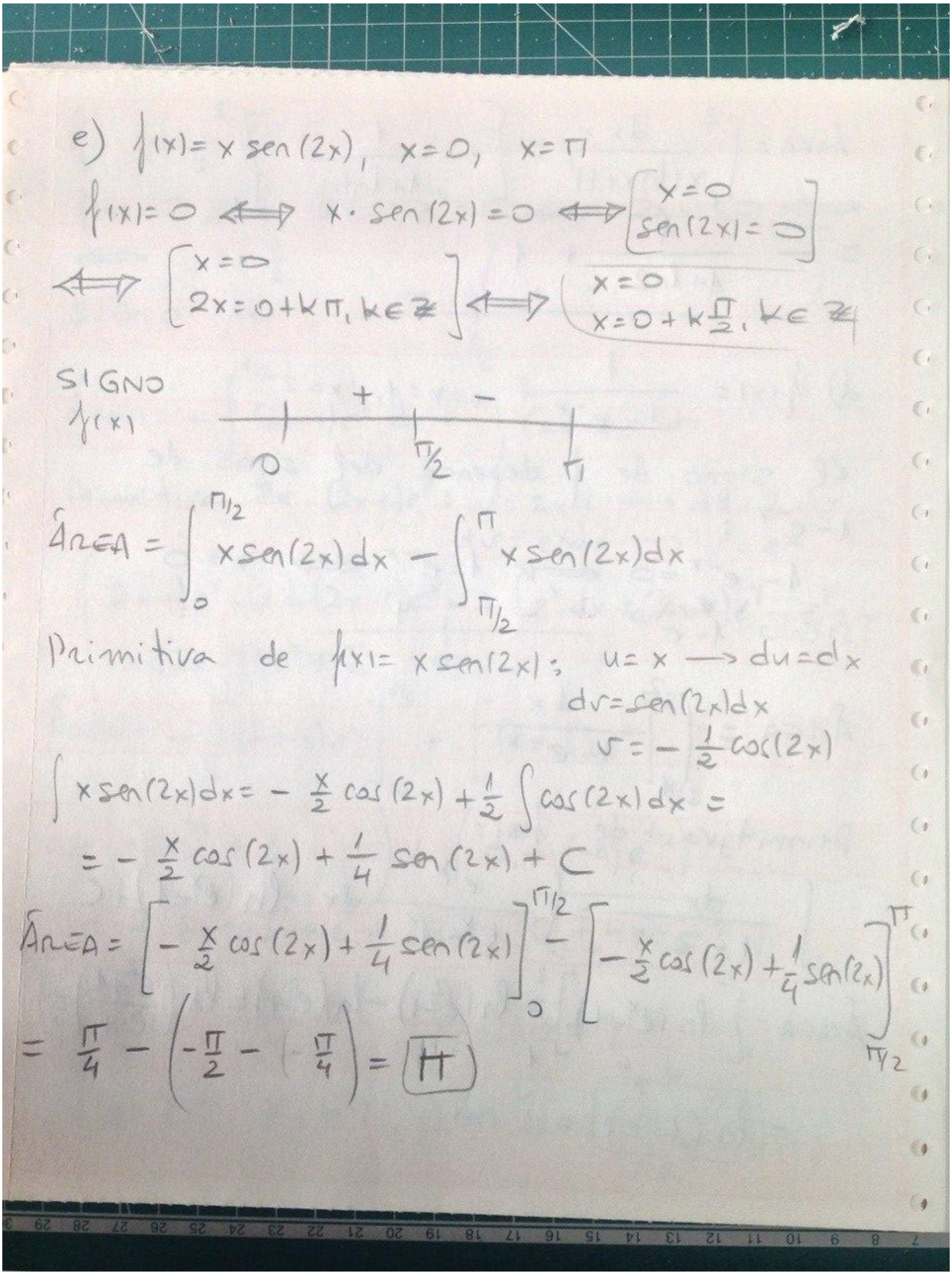
ÁREA $= \int_1^2 \frac{dx}{1-e^{-x}}$

Primitiva de $f(x)$:

$$\int \frac{dx}{1-e^{-x}} = \int \frac{e^x}{e^x-1}\,dx = \ln(e^x-1)+C$$

ÁREA $= \left[\ln(e^x-1)\right]_1^2 = \ln(e^2-1)-\ln(e-1) = \ln\left(\frac{e^2-1}{e-1}\right) =$

$= \boxed{\ln(1+e)}$



e) $f(x) = x \operatorname{sen}(2x)$, $x=0$, $x=\pi$

$$f(x)=0 \iff x \cdot \operatorname{sen}(2x) = 0 \iff \left[\begin{array}{l} x=0 \\ \operatorname{sen}(2x)=0 \end{array}\right]$$

$$\iff \left[\begin{array}{l} x=0 \\ 2x = 0 + k\pi,\ k\in\mathbb{Z} \end{array}\right] \iff \left(\begin{array}{l} x=0 \\ x = 0 + k\frac{\pi}{2},\ k\in\mathbb{Z} \end{array}\right.$$

SIGNO
$f(x)$

$$\begin{array}{ccccc} & + & & - & \\ \hline 0 & & \pi/2 & & \pi \end{array}$$

$$\text{ÁREA} = \int_0^{\pi/2} x \operatorname{sen}(2x)\,dx - \int_{\pi/2}^{\pi} x \operatorname{sen}(2x)\,dx$$

Primitiva de $f(x) = x\operatorname{sen}(2x)$: $u = x \longrightarrow du = dx$

$dv = \operatorname{sen}(2x)dx$

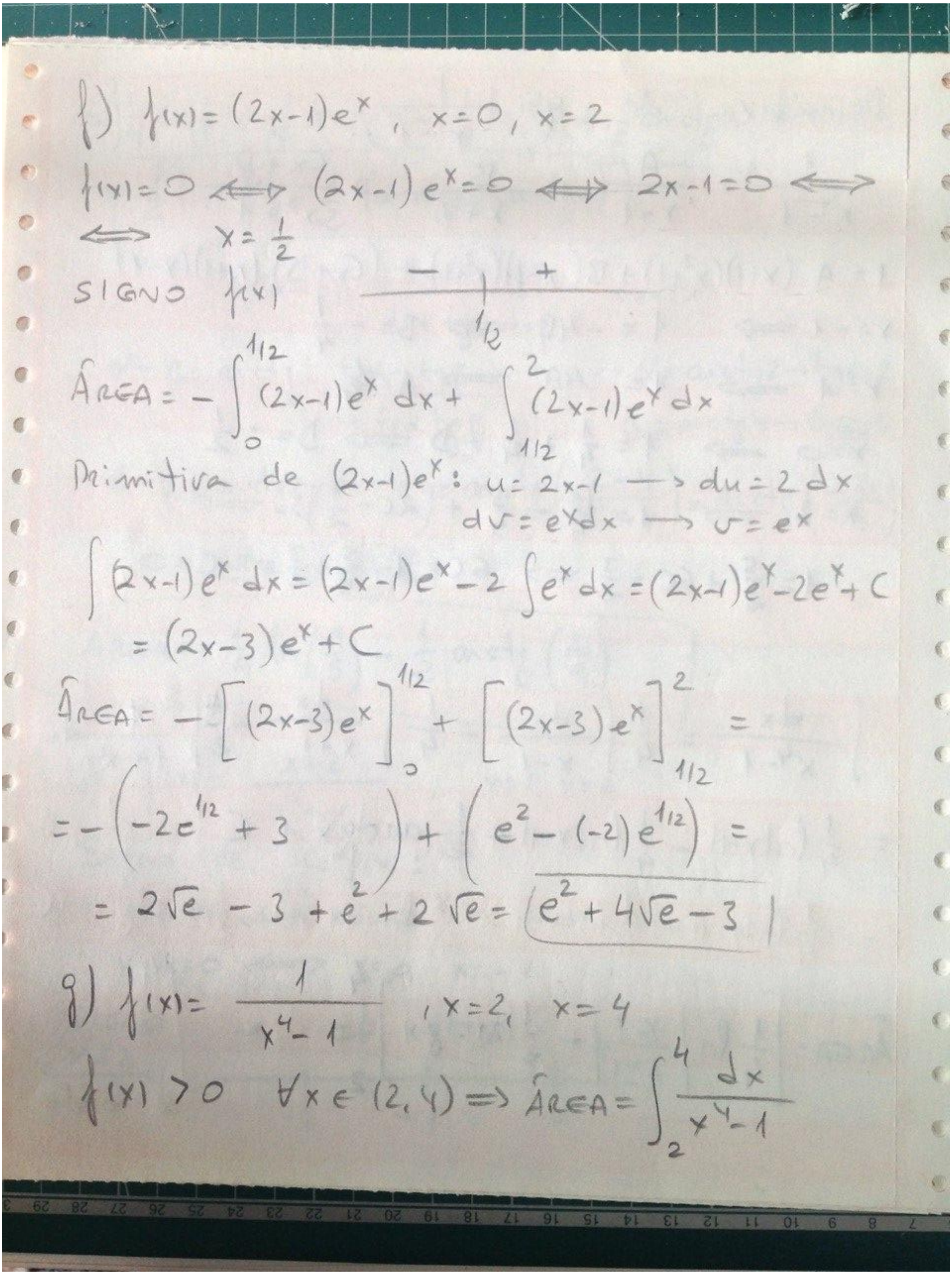
$v = -\frac{1}{2}\cos(2x)$

$$\int x\operatorname{sen}(2x)dx = -\frac{x}{2}\cos(2x) + \frac{1}{2}\int \cos(2x)dx =$$

$$= -\frac{x}{2}\cos(2x) + \frac{1}{4}\operatorname{sen}(2x) + C$$

$$\text{ÁREA} = \left[-\frac{x}{2}\cos(2x) + \frac{1}{4}\operatorname{sen}(2x)\right]_0^{\pi/2} - \left[-\frac{x}{2}\cos(2x) + \frac{1}{4}\operatorname{sen}(2x)\right]_{\pi/2}^{\pi}$$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{2} - \frac{\pi}{4}\right) = \boxed{\pi}$$



f) $f(x) = (2x-1)e^x \ , \ x=0, \ x=2$

$f(x)=0 \iff (2x-1)e^x=0 \iff 2x-1=0 \iff$

$\iff x = \frac{1}{2}$

SIGNO $f(x)$

$$\begin{array}{ccc} - & & + \\ \hline & \tfrac{1}{2} & \end{array}$$

$$\text{ÁREA} = -\int_0^{1/2} (2x-1)e^x \, dx + \int_{1/2}^{2} (2x-1)e^x \, dx$$

Primitiva de $(2x-1)e^x$: $u = 2x-1 \longrightarrow du = 2\,dx$

$dv = e^x dx \longrightarrow v = e^x$

$$\int (2x-1)e^x \, dx = (2x-1)e^x - 2\int e^x \, dx = (2x-1)e^x - 2e^x + C$$

$$= (2x-3)e^x + C$$

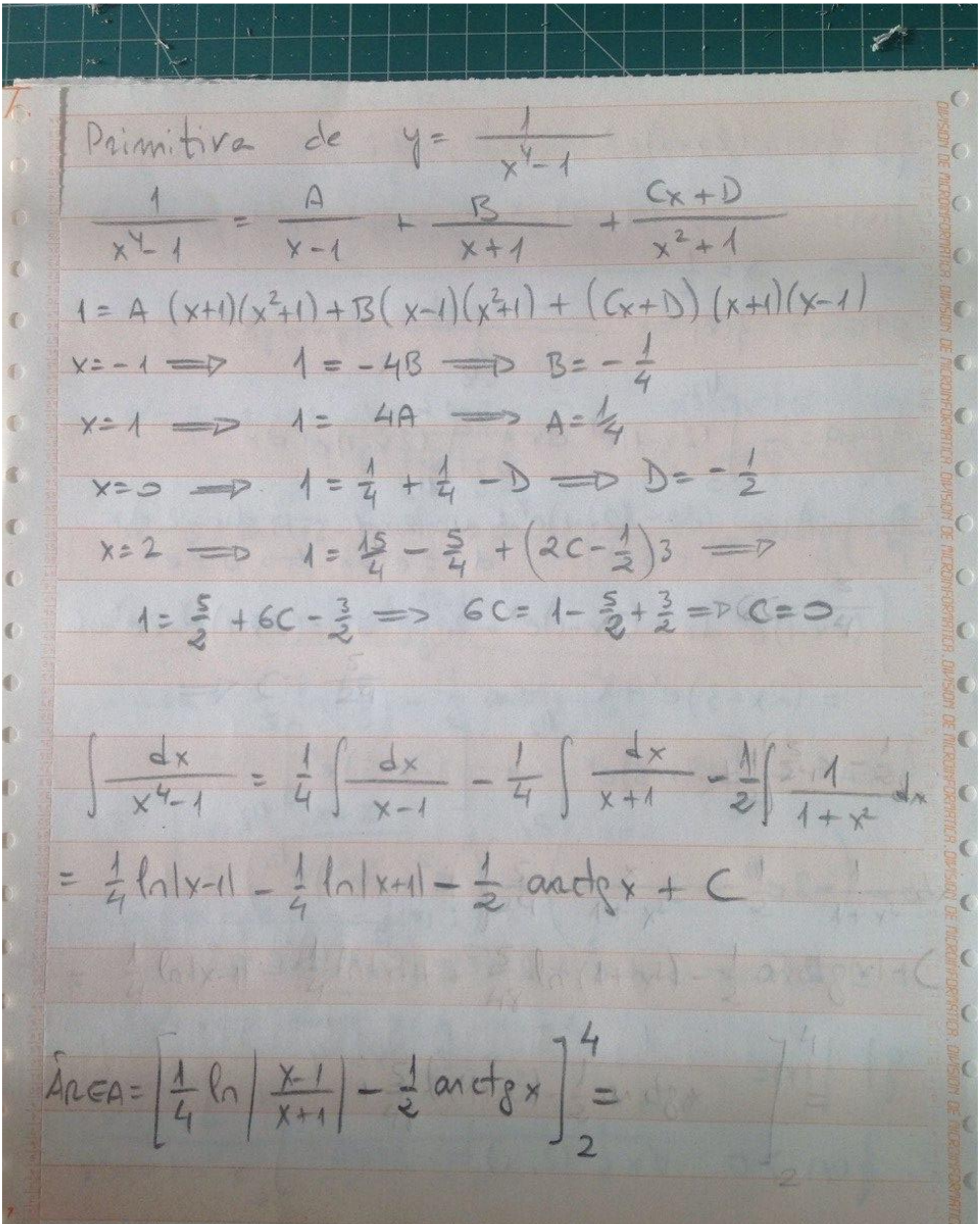
$$\text{ÁREA} = -\Big[(2x-3)e^x\Big]_0^{1/2} + \Big[(2x-3)e^x\Big]_{1/2}^{2} =$$

$$= -\left(-2e^{1/2} + 3\right) + \left(e^2 - (-2)e^{1/2}\right) =$$

$$= 2\sqrt{e} - 3 + e^2 + 2\sqrt{e} = \boxed{e^2 + 4\sqrt{e} - 3}$$

g) $f(x) = \dfrac{1}{x^4 - 1} \ , \ x=2, \ x=4$

$f(x) > 0 \quad \forall x \in (2,4) \Longrightarrow \text{ÁREA} = \displaystyle\int_2^4 \frac{dx}{x^4-1}$



Primitiva de $y = \frac{1}{x^4-1}$

$$\frac{1}{x^4-1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

$$1 = A\,(x+1)(x^2+1) + B(\,x-1)(x^2+1) + (Cx+D)\,(x+1)(x-1)$$

$$x=-1 \Longrightarrow 1 = -4B \Longrightarrow B = -\frac{1}{4}$$

$$x=1 \Longrightarrow 1 = 4A \Longrightarrow A = \frac{1}{4}$$

$$x=0 \Longrightarrow 1 = \frac{1}{4} + \frac{1}{4} - D \Longrightarrow D = -\frac{1}{2}$$

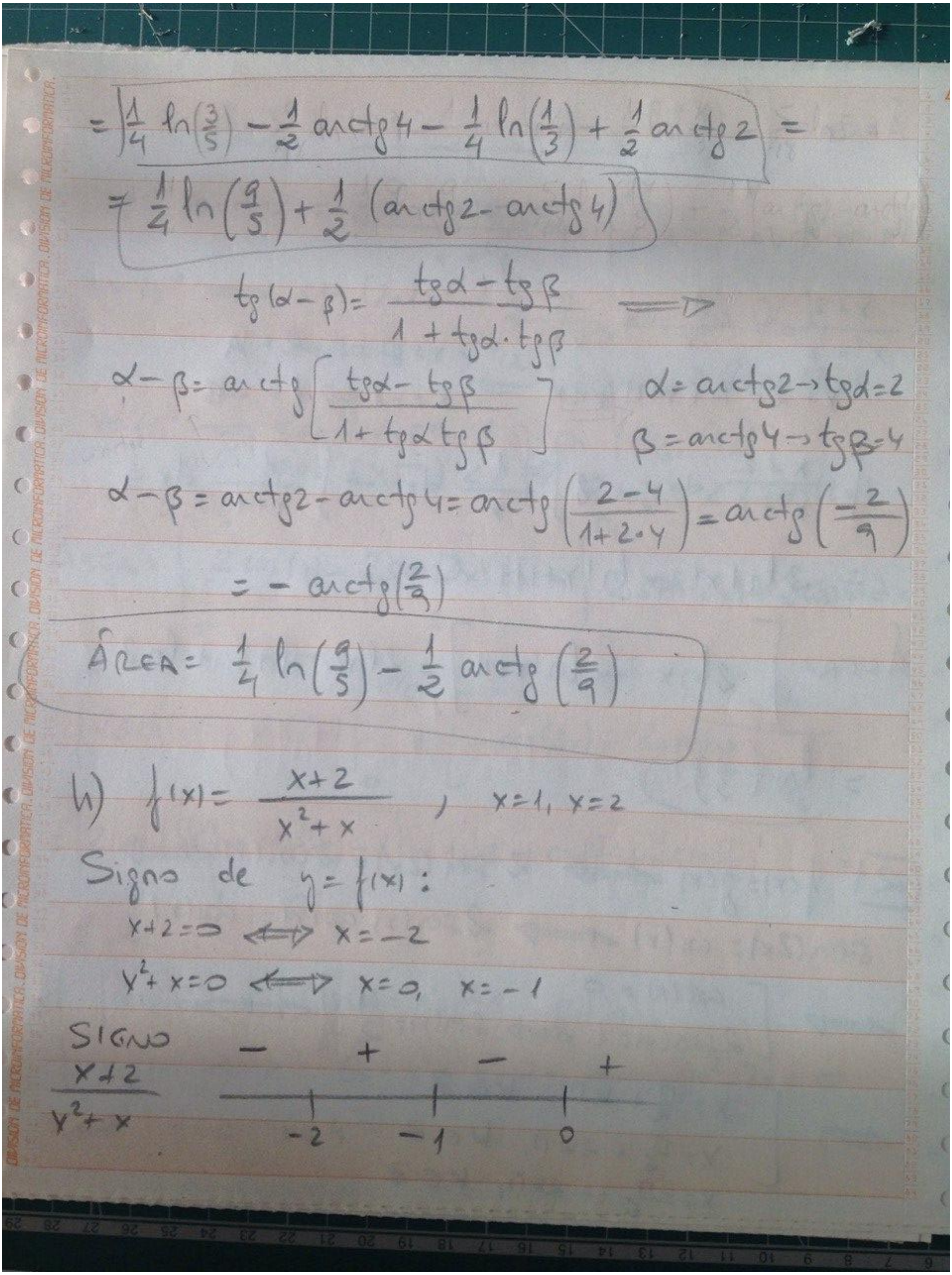
$$x=2 \Longrightarrow 1 = \frac{15}{4} - \frac{5}{4} + \left(2C - \frac{1}{2}\right)3 \Longrightarrow$$

$$1 = \frac{5}{2} + 6C - \frac{3}{2} \Longrightarrow 6C = 1 - \frac{5}{2} + \frac{3}{2} \Longrightarrow C = 0$$

$$\int \frac{dx}{x^4-1} = \frac{1}{4}\int \frac{dx}{x-1} - \frac{1}{4}\int \frac{dx}{x+1} - \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$

$$= \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| - \frac{1}{2}\operatorname{arctg} x + C$$

$$\text{ÁREA} = \left[\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\operatorname{arctg} x\right]_2^4 =$$



$$= \boxed{\frac{1}{4}\ln\left(\frac{3}{5}\right) - \frac{1}{2}\,\text{arctg}\,4 - \frac{1}{4}\ln\left(\frac{1}{3}\right) + \frac{1}{2}\,\text{arctg}\,2} =$$

$$= \boxed{\frac{1}{4}\ln\left(\frac{9}{5}\right) + \frac{1}{2}\left(\text{arctg}\,2 - \text{arctg}\,4\right)}$$

$$\text{tg}(\alpha-\beta) = \frac{\text{tg}\alpha - \text{tg}\beta}{1 + \text{tg}\alpha \cdot \text{tg}\beta} \Longrightarrow$$

$$\alpha - \beta = \text{arctg}\left[\frac{\text{tg}\alpha - \text{tg}\beta}{1 + \text{tg}\alpha\,\text{tg}\beta}\right] \qquad \begin{array}{l} \alpha = \text{arctg}\,2 \to \text{tg}\alpha = 2 \\ \beta = \text{arctg}\,4 \to \text{tg}\beta = 4 \end{array}$$

$$\alpha - \beta = \text{arctg}\,2 - \text{arctg}\,4 = \text{arctg}\left(\frac{2-4}{1+2\cdot 4}\right) = \text{arctg}\left(\frac{-2}{9}\right)$$

$$= -\,\text{arctg}\left(\frac{2}{9}\right)$$

$$\boxed{\text{ÁREA} = \frac{1}{4}\ln\left(\frac{9}{5}\right) - \frac{1}{2}\,\text{arctg}\left(\frac{2}{9}\right)}$$

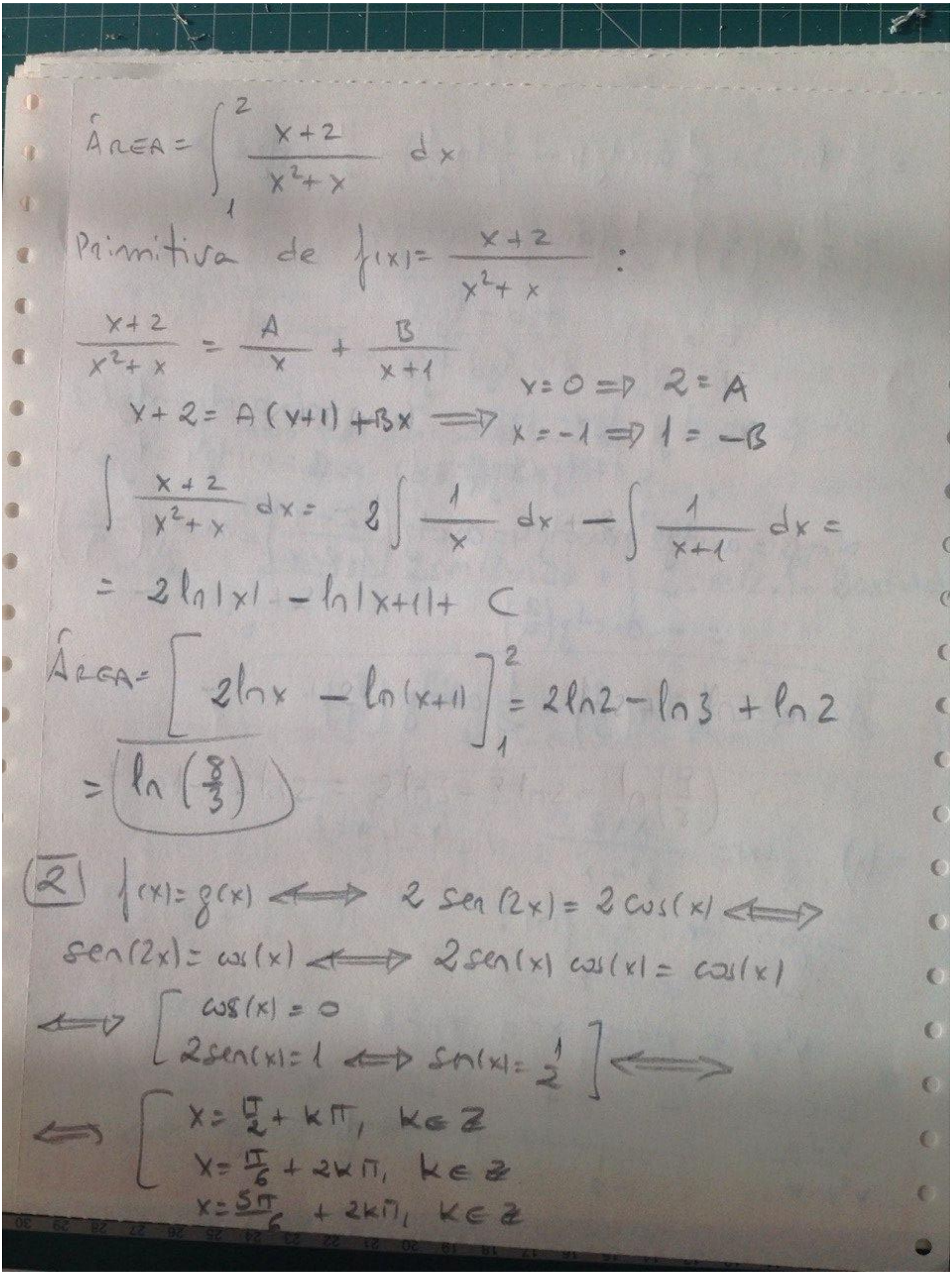
h) $f(x) = \dfrac{x+2}{x^2+x}$, $x=1$, $x=2$

Signo de $y = f(x)$:

$x+2=0 \Longleftrightarrow x=-2$

$x^2+x=0 \Longleftrightarrow x=0,\ x=-1$

$$\begin{array}{lccccccc} \text{SIGNO} & - & & + & & - & & + \\ \frac{x+2}{x^2+x} & & | & & | & & | & \\ & & -2 & & -1 & & 0 & \end{array}$$



$$\text{Área} = \int_1^2 \frac{x+2}{x^2+x}\,dx$$

Primitiva de $f(x) = \dfrac{x+2}{x^2+x}$:

$$\frac{x+2}{x^2+x} = \frac{A}{x} + \frac{B}{x+1}$$

$$x+2 = A(x+1) + Bx \Longrightarrow \begin{array}{l} x=0 \Rightarrow 2 = A \\ x=-1 \Rightarrow 1 = -B \end{array}$$

$$\int \frac{x+2}{x^2+x}\,dx = 2\int \frac{1}{x}\,dx - \int \frac{1}{x+1}\,dx =$$

$$= 2\ln|x| - \ln|x+1| + C$$

$$\text{Área} = \Big[2\ln x - \ln(x+1)\Big]_1^2 = 2\ln 2 - \ln 3 + \ln 2$$

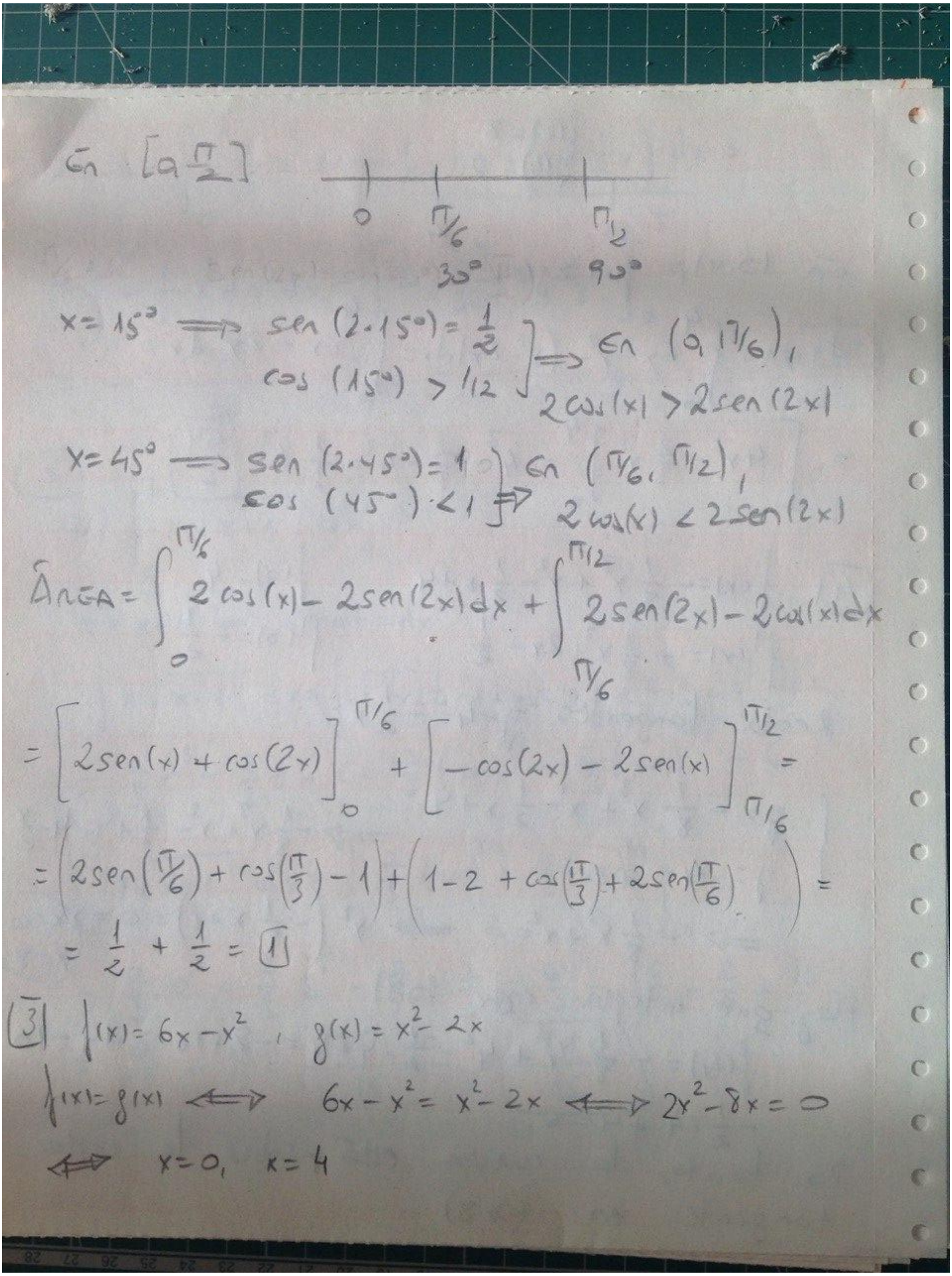
$$= \boxed{\ln\left(\frac{8}{3}\right)}$$

$\boxed{2}$ $f(x) = g(x) \Longleftrightarrow 2\,\text{sen}(2x) = 2\cos(x) \Longleftrightarrow$

$\text{sen}(2x) = \cos(x) \Longleftrightarrow 2\,\text{sen}(x)\cos(x) = \cos(x)$

$$\Longleftrightarrow \left[\begin{array}{l} \cos(x) = 0 \\ 2\,\text{sen}(x) = 1 \Longleftrightarrow \text{sen}(x) = \dfrac{1}{2} \end{array}\right] \Longleftrightarrow$$

$$\Longleftrightarrow \left[\begin{array}{l} x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z} \\ x = \frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z} \\ x = \frac{5\pi}{6} + 2k\pi,\ k \in \mathbb{Z} \end{array}\right.$$



En $\left[0, \frac{\pi}{2}\right]$

$$x = 15^\circ \Longrightarrow \left.\begin{array}{l} \text{sen}(2\cdot 15^\circ) = \frac{1}{2} \\ \cos(15^\circ) > 1/2 \end{array}\right] \Longrightarrow \text{En } (0, \pi/6), \; 2\cos(x) > 2\,\text{sen}(2x)$$

$$x = 45^\circ \Longrightarrow \left.\begin{array}{l} \text{sen}(2\cdot 45^\circ) = 1 \\ \cos(45^\circ) < 1 \end{array}\right] \Longrightarrow \text{En } (\pi/6, \pi/2), \; 2\cos(x) < 2\,\text{sen}(2x)$$

$$\text{Área} = \int_0^{\pi/6} 2\cos(x) - 2\,\text{sen}(2x)\,dx + \int_{\pi/6}^{\pi/2} 2\,\text{sen}(2x) - 2\cos(x)\,dx$$

$$= \Big[2\,\text{sen}(x) + \cos(2x)\Big]_0^{\pi/6} + \Big[-\cos(2x) - 2\,\text{sen}(x)\Big]_{\pi/6}^{\pi/2} =$$

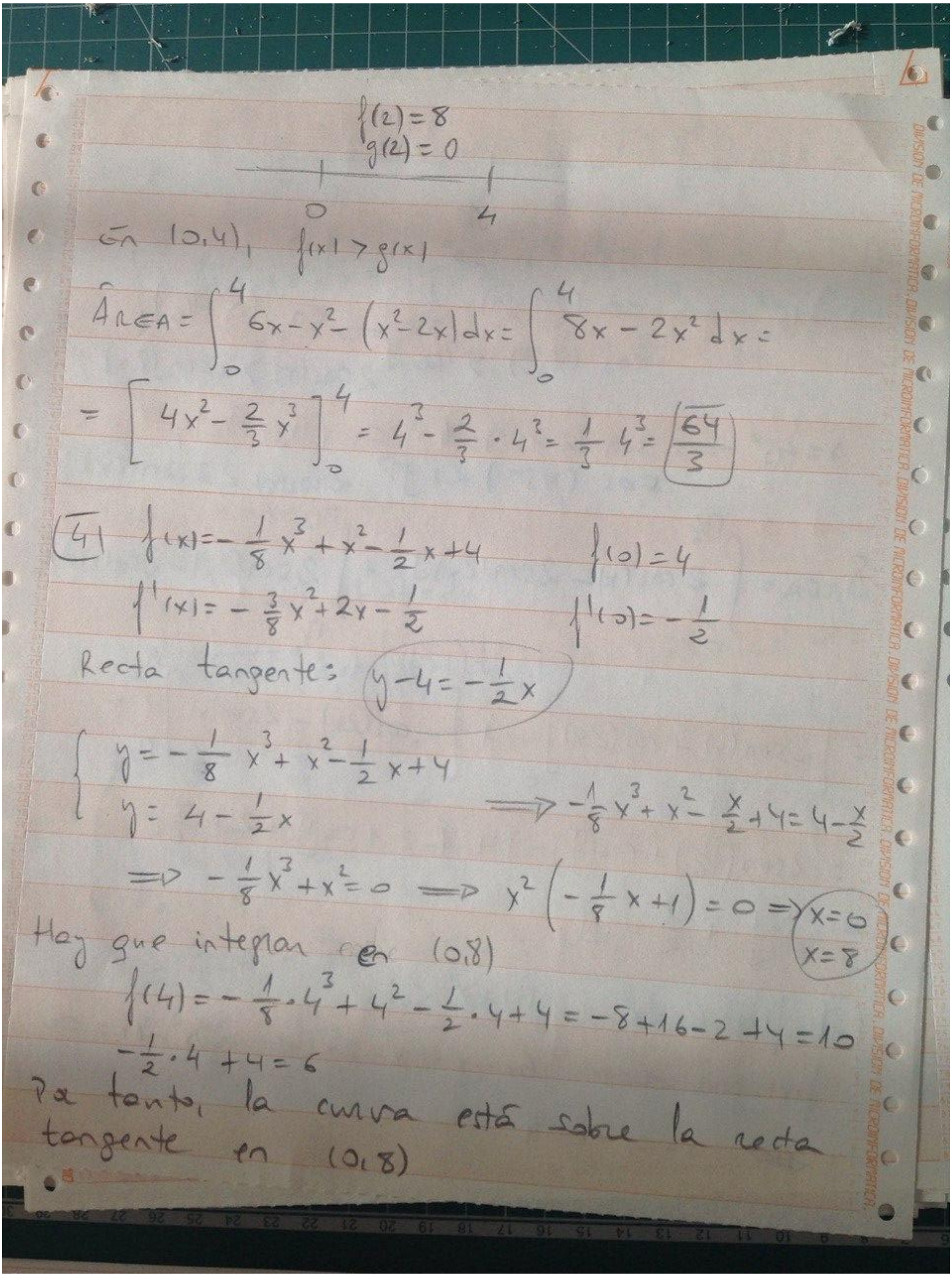
$$= \left(2\,\text{sen}\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{3}\right) - 1\right) + \left(1 - 2 + \cos\left(\frac{\pi}{3}\right) + 2\,\text{sen}\left(\frac{\pi}{6}\right)\right) =$$

$$= \frac{1}{2} + \frac{1}{2} = \boxed{1}$$

$\boxed{3}$ $f(x) = 6x - x^2$, $g(x) = x^2 - 2x$

$$f(x) = g(x) \Longleftrightarrow 6x - x^2 = x^2 - 2x \Longleftrightarrow 2x^2 - 8x = 0$$

$$\Longleftrightarrow x = 0, \quad x = 4$$



$$\begin{cases} f(2)=8 \\ g(2)=0 \end{cases}$$

En $(0,4)$, $f(x) > g(x)$

$$\text{ÁREA} = \int_0^4 6x - x^2 - (x^2 - 2x)\,dx = \int_0^4 8x - 2x^2\,dx =$$

$$= \left[4x^2 - \frac{2}{3}x^3\right]_0^4 = 4^3 - \frac{2}{3}\cdot 4^3 = \frac{1}{3}4^3 = \boxed{\frac{64}{3}}$$

$\boxed{4}$ $f(x) = -\frac{1}{8}x^3 + x^2 - \frac{1}{2}x + 4$ $\qquad f(0) = 4$

$f'(x) = -\frac{3}{8}x^2 + 2x - \frac{1}{2}$ $\qquad f'(0) = -\frac{1}{2}$

Recta tangente: $y - 4 = -\frac{1}{2}x$

$$\begin{cases} y = -\frac{1}{8}x^3 + x^2 - \frac{1}{2}x + 4 \\ y = 4 - \frac{1}{2}x \end{cases} \Longrightarrow -\frac{1}{8}x^3 + x^2 - \frac{x}{2} + 4 = 4 - \frac{x}{2}$$

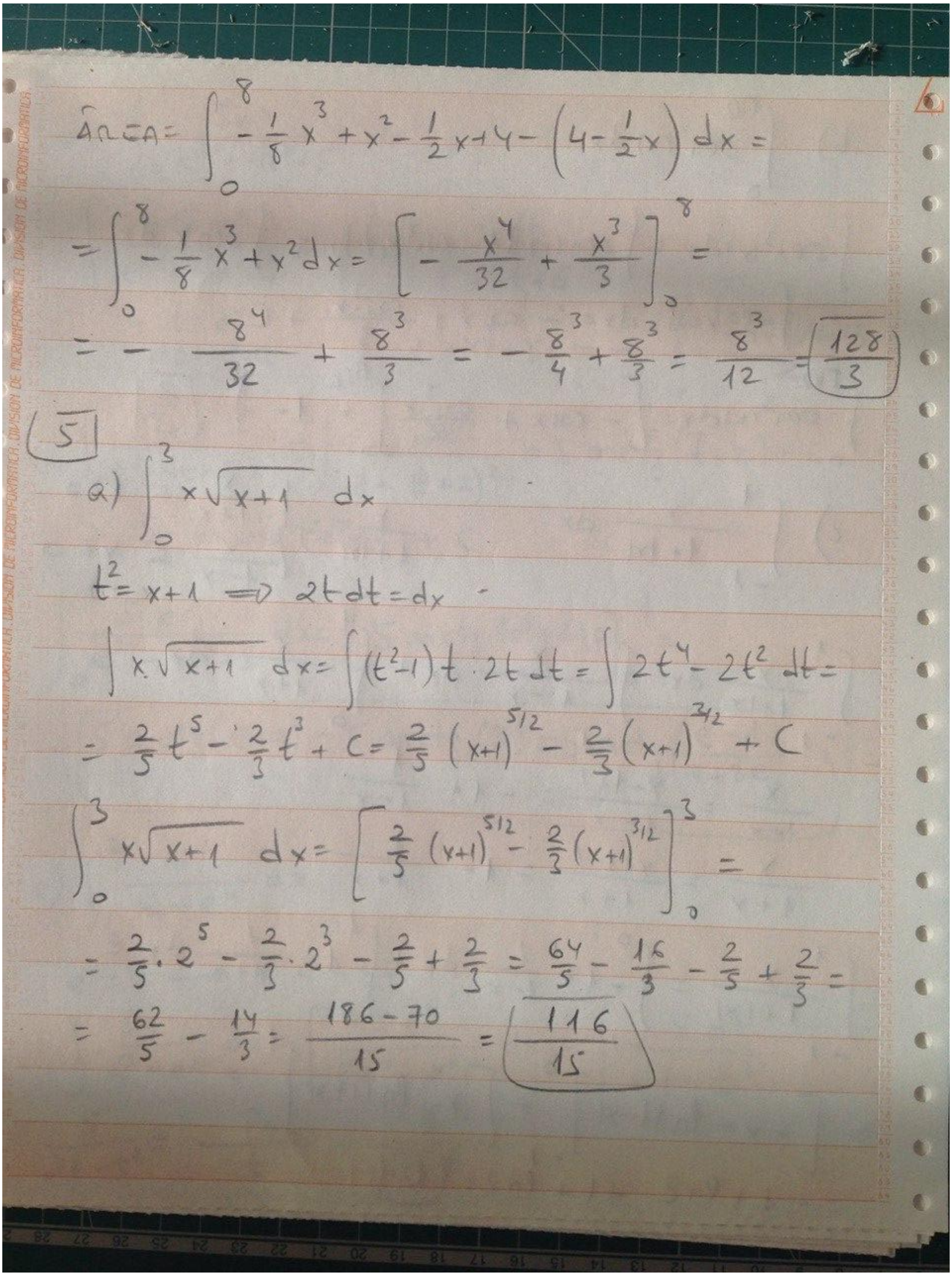
$$\Longrightarrow -\frac{1}{8}x^3 + x^2 = 0 \Longrightarrow x^2\left(-\frac{1}{8}x + 1\right) = 0 \Longrightarrow \begin{matrix} x=0 \\ x=8 \end{matrix}$$

Hay que integrar en $(0,8)$

$f(4) = -\frac{1}{8}\cdot 4^3 + 4^2 - \frac{1}{2}\cdot 4 + 4 = -8 + 16 - 2 + 4 = 10$

$-\frac{1}{2}\cdot 4 + 4 = 6$

Por tanto, la curva está sobre la recta tangente en $(0,8)$



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ÁREA $= \int_0^8 -\frac{1}{8}x^3 + x^2 - \frac{1}{2}x + 4 - \left(4 - \frac{1}{2}x\right) dx =$

$$= \int_0^8 -\frac{1}{8}x^3 + x^2 dx = \left[-\frac{x^4}{32} + \frac{x^3}{3}\right]_0^8 =$$

$$= -\frac{8^4}{32} + \frac{8^3}{3} = -\frac{8^3}{4} + \frac{8^3}{3} = \frac{8^3}{12} = \boxed{\frac{128}{3}}$$

$\boxed{5}$

a) $\int_0^3 x\sqrt{x+1}\;dx$

$t^2 = x+1 \Longrightarrow 2t\,dt = dx$

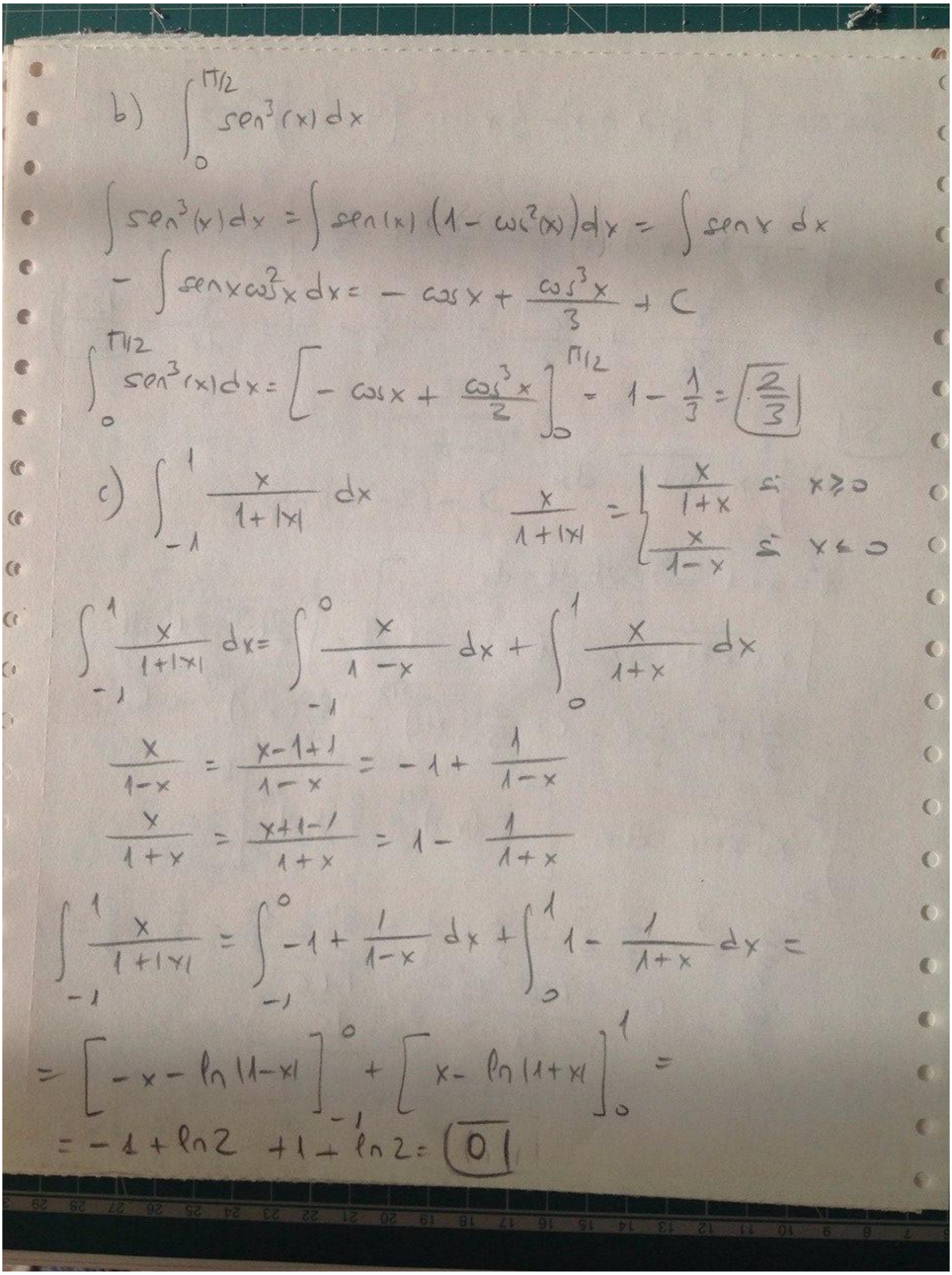
$$\int x\sqrt{x+1}\;dx = \int (t^2-1)t\cdot 2t\,dt = \int 2t^4 - 2t^2\,dt =$$

$$= \frac{2}{5}t^5 - \frac{2}{3}t^3 + C = \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$$

$$\int_0^3 x\sqrt{x+1}\;dx = \left[\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2}\right]_0^3 =$$

$$= \frac{2}{5}\cdot 2^5 - \frac{2}{3}\cdot 2^3 - \frac{2}{5} + \frac{2}{3} = \frac{64}{5} - \frac{16}{3} - \frac{2}{5} + \frac{2}{3} =$$

$$= \frac{62}{5} - \frac{14}{3} = \frac{186-70}{15} = \boxed{\frac{116}{15}}$$



b) $\int_0^{\pi/2} sen^3(x)\,dx$

$$\int sen^3(x)\,dx = \int sen(x)\left(1-\cos^2(x)\right)dx = \int sen\,x\,dx - \int sen\,x\cos^2 x\,dx = -\cos x + \frac{\cos^3 x}{3} + C$$

$$\int_0^{\pi/2} sen^3(x)\,dx = \left[-\cos x + \frac{\cos^3 x}{2}\right]_0^{\pi/2} = 1 - \frac{1}{3} = \boxed{\frac{2}{3}}$$

c) $\int_{-1}^{1} \frac{x}{1+|x|}\,dx$

$$\frac{x}{1+|x|} = \begin{cases} \frac{x}{1+x} & si\ x \geq 0 \\ \frac{x}{1-x} & si\ x < 0 \end{cases}$$

$$\int_{-1}^{1} \frac{x}{1+|x|}\,dx = \int_{-1}^{0} \frac{x}{1-x}\,dx + \int_{0}^{1} \frac{x}{1+x}\,dx$$

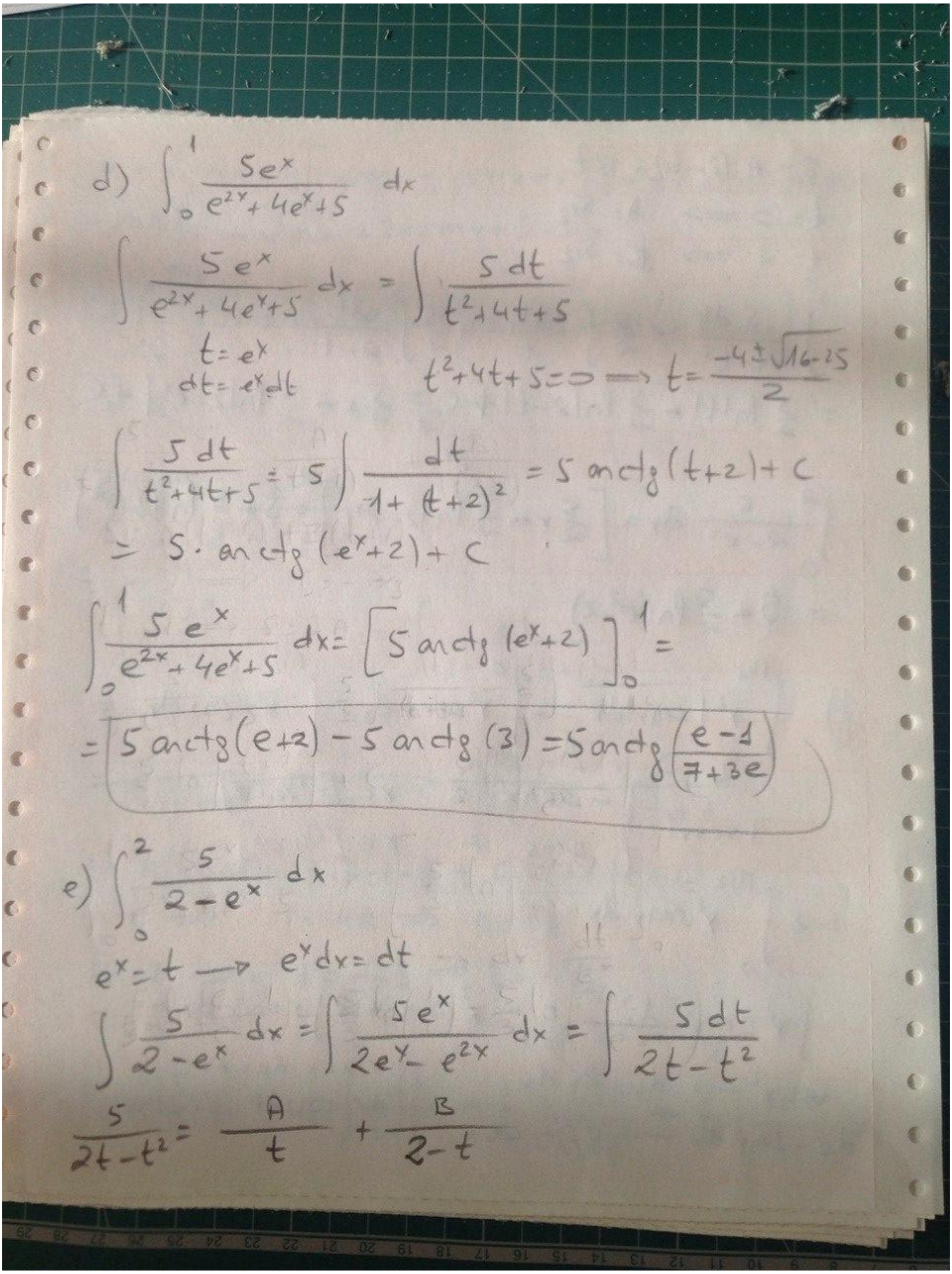
$$\frac{x}{1-x} = \frac{x-1+1}{1-x} = -1 + \frac{1}{1-x}$$

$$\frac{x}{1+x} = \frac{x+1-1}{1+x} = 1 - \frac{1}{1+x}$$

$$\int_{-1}^{1} \frac{x}{1+|x|} = \int_{-1}^{0} -1 + \frac{1}{1-x}\,dx + \int_{0}^{1} 1 - \frac{1}{1+x}\,dx =$$

$$= \left[-x - \ln|1-x|\right]_{-1}^{0} + \left[x - \ln|1+x|\right]_{0}^{1} =$$

$$= -1 + \ln 2 + 1 - \ln 2 = \boxed{0}$$



d) $\int_0^1 \frac{5e^x}{e^{2x}+4e^x+5}\,dx$

$$\int \frac{5e^x}{e^{2x}+4e^x+5}\,dx = \int \frac{5\,dt}{t^2+4t+5}$$

$t = e^x$

$dt = e^x dt$

$$t^2+4t+5=0 \Longrightarrow t=\frac{-4\pm\sqrt{16-25}}{2}$$

$$\int \frac{5\,dt}{t^2+4t+5} = 5\int \frac{dt}{1+(t+2)^2} = 5\,\mathrm{arctg}(t+2)+C$$

$$= 5\cdot \mathrm{arctg}(e^x+2)+C$$

$$\int_0^1 \frac{5e^x}{e^{2x}+4e^x+5}\,dx = \left[5\,\mathrm{arctg}(e^x+2)\right]_0^1 =$$

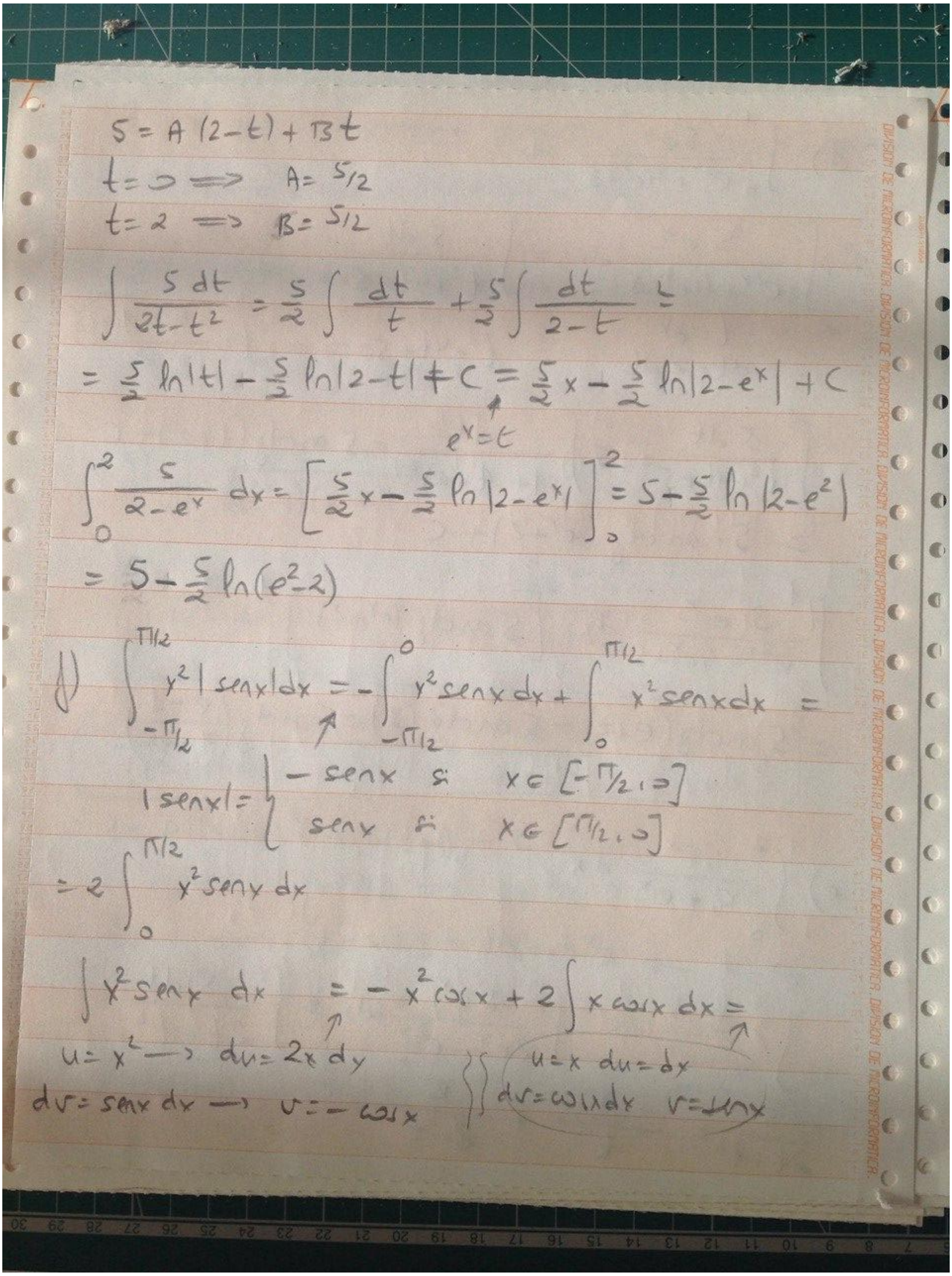
$$= \boxed{5\,\mathrm{arctg}(e+2) - 5\,\mathrm{arctg}(3) = 5\,\mathrm{arctg}\left(\frac{e-1}{7+3e}\right)}$$

e) $\int_0^2 \frac{5}{2-e^x}\,dx$

$e^x = t \longrightarrow e^x dx = dt$

$$\int \frac{5}{2-e^x}\,dx = \int \frac{5e^x}{2e^x-e^{2x}}\,dx = \int \frac{5\,dt}{2t-t^2}$$

$$\frac{5}{2t-t^2} = \frac{A}{t} + \frac{B}{2-t}$$



$5 = A(2-t) + Bt$

$t = 0 \implies A = 5/2$

$t = 2 \implies B = 5/2$

$$\int \frac{5\,dt}{2t-t^2} = \frac{5}{2}\int \frac{dt}{t} + \frac{5}{2}\int \frac{dt}{2-t} =$$

$$= \frac{5}{2}\ln|t| - \frac{5}{2}\ln|2-t| + C \underset{\overset{\uparrow}{e^x=t}}{=} \frac{5}{2}x - \frac{5}{2}\ln|2-e^x| + C$$

$$\int_0^2 \frac{5}{2-e^x}\,dx = \left[\frac{5}{2}x - \frac{5}{2}\ln|2-e^x|\right]_0^2 = 5 - \frac{5}{2}\ln|2-e^2|$$

$$= 5 - \frac{5}{2}\ln(e^2-2)$$

f) $$\int_{-\pi/2}^{\pi/2} x^2|\operatorname{sen}x|\,dx \underset{\uparrow}{=} -\int_{-\pi/2}^{0} x^2\operatorname{sen}x\,dx + \int_0^{\pi/2} x^2\operatorname{sen}x\,dx =$$

$$|\operatorname{sen}x| = \begin{cases} -\operatorname{sen}x & \text{si } x \in [-\pi/2, 0] \\ \operatorname{sen}x & \text{si } x \in [\pi/2, 0] \end{cases}$$

$$= 2\int_0^{\pi/2} x^2\operatorname{sen}x\,dx$$

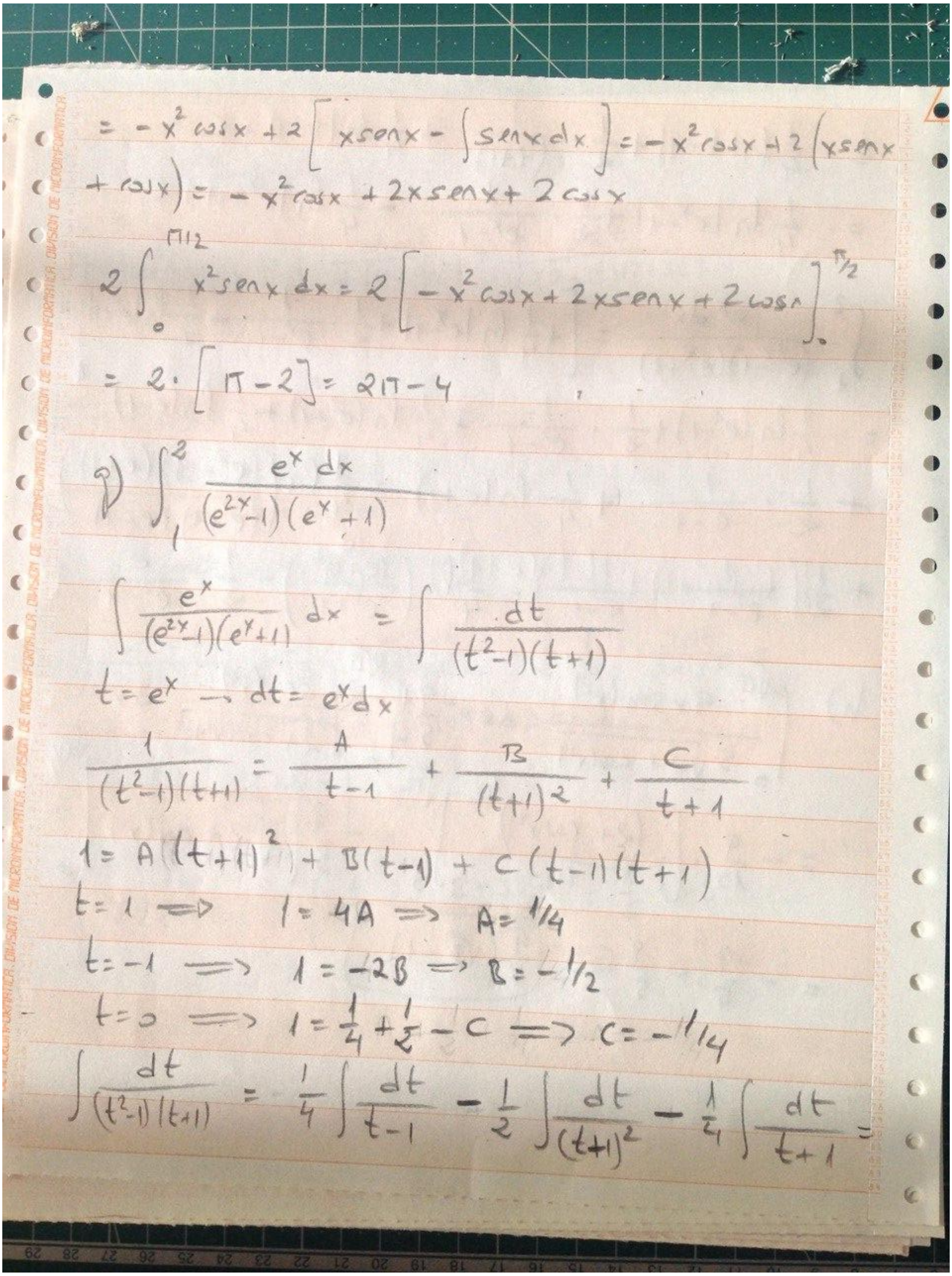
$$\int x^2\operatorname{sen}x\,dx \underset{\uparrow}{=} -x^2\cos x + 2\int x\cos x\,dx \underset{\uparrow}{=}$$

$u = x^2 \longrightarrow du = 2x\,dx$

$dv = \operatorname{sen}x\,dx \longrightarrow v = -\cos x$

$u = x \quad du = dx$

$dv = \cos x\,dx \quad v = \operatorname{sen}x$



$$= -x^2\cos x + 2\left[x\,\text{sen}x - \int \text{sen}x\,dx\right] = -x^2\cos x + 2\left(x\,\text{sen}x + \cos x\right) = -x^2\cos x + 2x\,\text{sen}x + 2\cos x$$

$$2\int_0^{\pi/2} x^2\,\text{sen}x\,dx = 2\left[-x^2\cos x + 2x\,\text{sen}x + 2\cos x\right]_0^{\pi/2}$$

$$= 2\cdot\left[\pi - 2\right] = 2\pi - 4$$

g) $\displaystyle\int_1^2 \frac{e^x\,dx}{(e^{2x}-1)(e^x+1)}$

$$\int \frac{e^x}{(e^{2x}-1)(e^x+1)}\,dx = \int \frac{dt}{(t^2-1)(t+1)}$$

$t = e^x \rightarrow dt = e^x dx$

$$\frac{1}{(t^2-1)(t+1)} = \frac{A}{t-1} + \frac{B}{(t+1)^2} + \frac{C}{t+1}$$

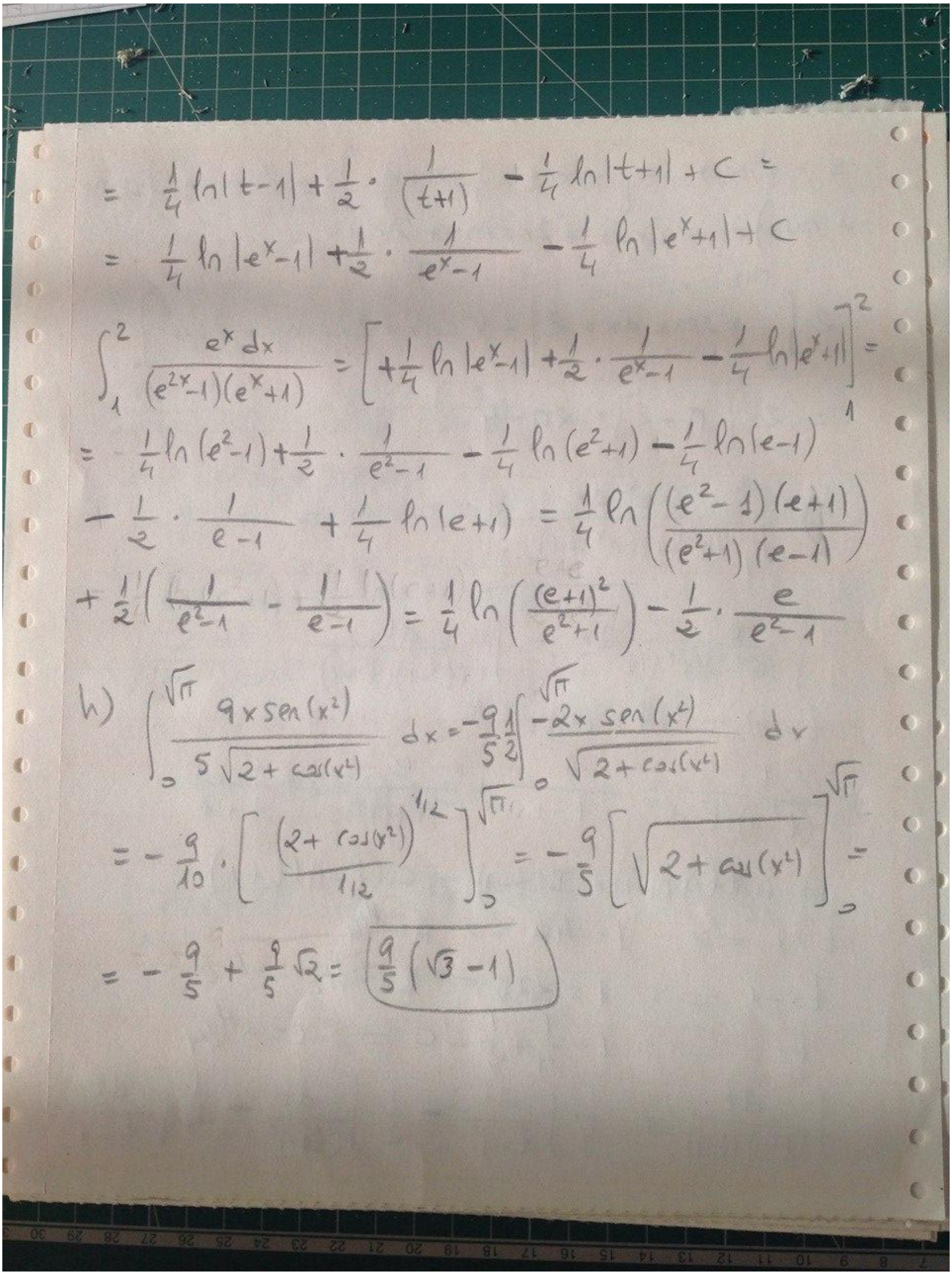
$1 = A(t+1)^2 + B(t-1) + C(t-1)(t+1)$

$t = 1 \Longrightarrow 1 = 4A \Longrightarrow A = 1/4$

$t = -1 \Longrightarrow 1 = -2B \Longrightarrow B = -1/2$

$t = 0 \Longrightarrow 1 = \frac{1}{4} + \frac{1}{2} - C \Longrightarrow C = -1/4$

$$\int \frac{dt}{(t^2-1)(t+1)} = \frac{1}{4}\int\frac{dt}{t-1} - \frac{1}{2}\int\frac{dt}{(t+1)^2} - \frac{1}{4}\int\frac{dt}{t+1} =$$



$$= \frac{1}{4}\ln|t-1| + \frac{1}{2}\cdot\frac{1}{(t+1)} - \frac{1}{4}\ln|t+1| + C =$$

$$= \frac{1}{4}\ln|e^x-1| + \frac{1}{2}\cdot\frac{1}{e^x-1} - \frac{1}{4}\ln|e^x+1| + C$$

$$\int_1^2 \frac{e^x\,dx}{(e^{2x}-1)(e^x+1)} = \left[+\frac{1}{4}\ln|e^x-1| + \frac{1}{2}\cdot\frac{1}{e^x-1} - \frac{1}{4}\ln|e^x+1|\right]_1^2 =$$

$$= \frac{1}{4}\ln(e^2-1) + \frac{1}{2}\cdot\frac{1}{e^2-1} - \frac{1}{4}\ln(e^2+1) - \frac{1}{4}\ln(e-1)$$

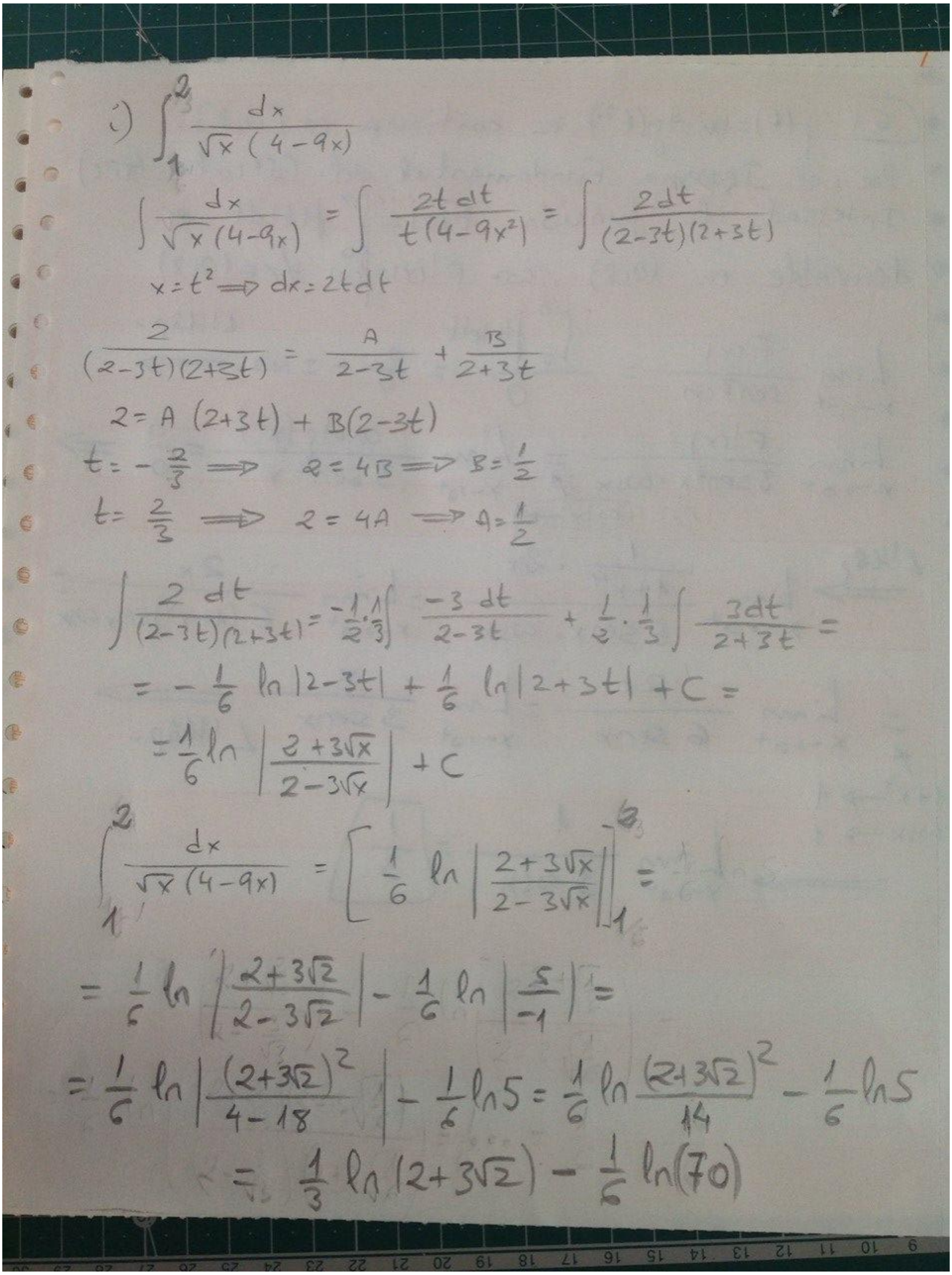
$$-\frac{1}{2}\cdot\frac{1}{e-1} + \frac{1}{4}\ln(e+1) = \frac{1}{4}\ln\left(\frac{(e^2-1)(e+1)}{(e^2+1)(e-1)}\right)$$

$$+\frac{1}{2}\left(\frac{1}{e^2-1} - \frac{1}{e-1}\right) = \frac{1}{4}\ln\left(\frac{(e+1)^2}{e^2+1}\right) - \frac{1}{2}\cdot\frac{e}{e^2-1}$$

h) $\displaystyle\int_0^{\sqrt{\pi}} \frac{9x\,sen(x^2)}{5\sqrt{2+\cos(x^2)}}\,dx = -\frac{9}{5}\frac{1}{2}\int_0^{\sqrt{\pi}} \frac{-2x\,sen(x^2)}{\sqrt{2+\cos(x^2)}}\,dx$

$$= -\frac{9}{10}\cdot\left[\frac{\left(2+\cos(x^2)\right)^{1/2}}{1/2}\right]_0^{\sqrt{\pi}} = -\frac{9}{5}\left[\sqrt{2+\cos(x^2)}\right]_0^{\sqrt{\pi}} =$$

$$= -\frac{9}{5} + \frac{9}{5}\sqrt{2} = \boxed{\frac{9}{5}\left(\sqrt{3}-1\right)}$$



i) $\int_1^2 \frac{dx}{\sqrt{x}\,(4-9x)}$

$$\int \frac{dx}{\sqrt{x}\,(4-9x)} = \int \frac{2t\,dt}{t(4-9x^2)} = \int \frac{2dt}{(2-3t)(2+3t)}$$

$x = t^2 \Longrightarrow dx = 2t\,dt$

$$\frac{2}{(2-3t)(2+3t)} = \frac{A}{2-3t} + \frac{B}{2+3t}$$

$2 = A(2+3t) + B(2-3t)$

$t = -\frac{2}{3} \Longrightarrow 2 = 4B \Longrightarrow B = \frac{1}{2}$

$t = \frac{2}{3} \Longrightarrow 2 = 4A \Longrightarrow A = \frac{1}{2}$

$$\int \frac{2\,dt}{(2-3t)(2+3t)} = -\frac{1}{2}\cdot\frac{1}{3}\int \frac{-3\,dt}{2-3t} + \frac{1}{2}\cdot\frac{1}{3}\int \frac{3dt}{2+3t} =$$

$$= -\frac{1}{6}\ln|2-3t| + \frac{1}{6}\ln|2+3t| + C =$$

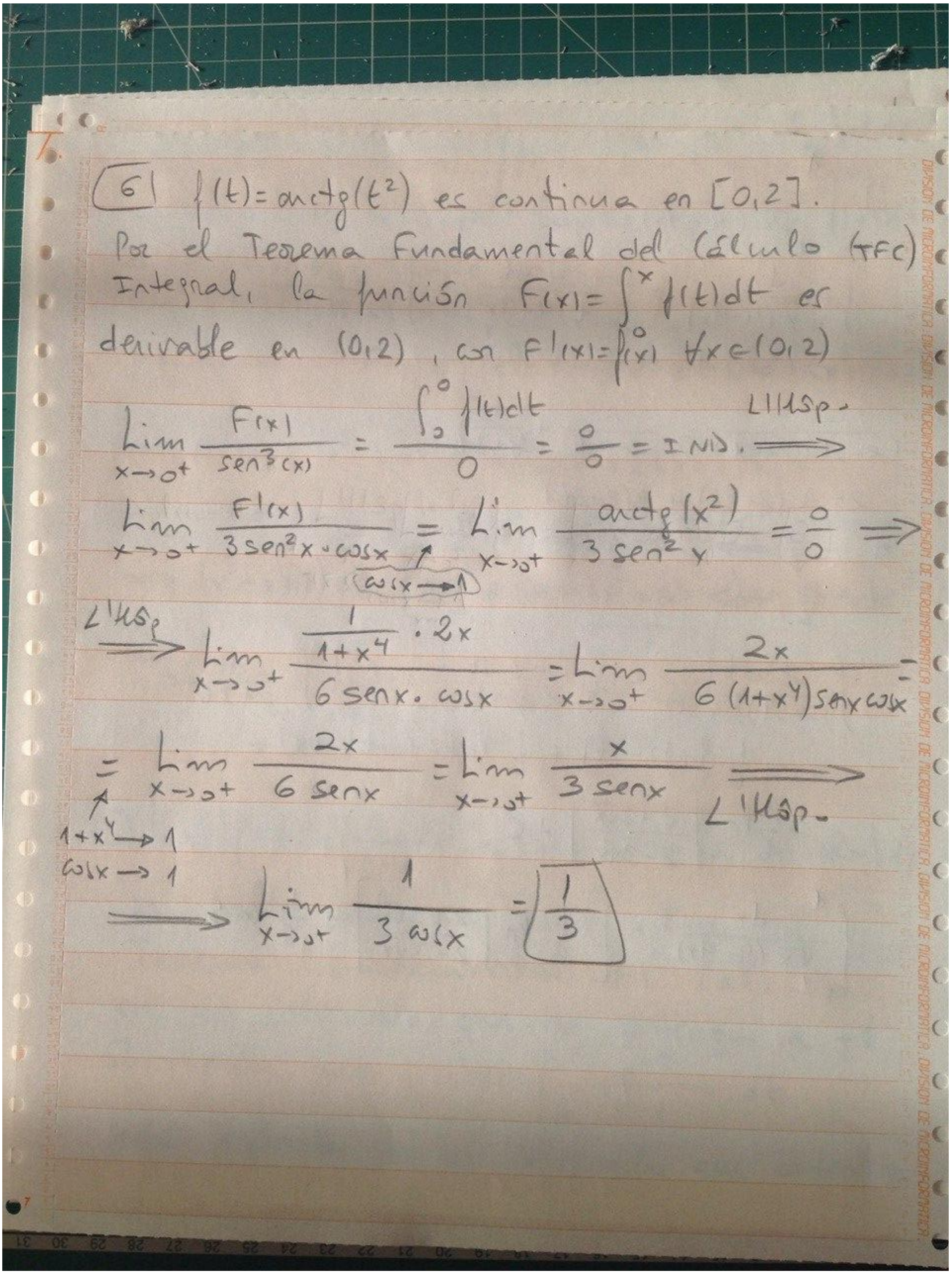
$$= \frac{1}{6}\ln\left|\frac{2+3\sqrt{x}}{2-3\sqrt{x}}\right| + C$$

$$\int_1^2 \frac{dx}{\sqrt{x}\,(4-9x)} = \left[\frac{1}{6}\ln\left|\frac{2+3\sqrt{x}}{2-3\sqrt{x}}\right|\right]_1^2 =$$

$$= \frac{1}{6}\ln\left|\frac{2+3\sqrt{2}}{2-3\sqrt{2}}\right| - \frac{1}{6}\ln\left|\frac{5}{-1}\right| =$$

$$= \frac{1}{6}\ln\left|\frac{(2+3\sqrt{2})^2}{4-18}\right| - \frac{1}{6}\ln 5 = \frac{1}{6}\ln\frac{(2+3\sqrt{2})^2}{14} - \frac{1}{6}\ln 5$$

$$= \frac{1}{3}\ln(2+3\sqrt{2}) - \frac{1}{6}\ln(70)$$



7.

(6) $f(t) = \text{arctg}(t^2)$ es continua en $[0,2]$.

Por el Teorema Fundamental del Cálculo (TFC) Integral, la función $F(x) = \int_0^x f(t)\,dt$ es derivable en $(0,2)$, con $F'(x) = f(x)$ $\forall x \in (0,2)$

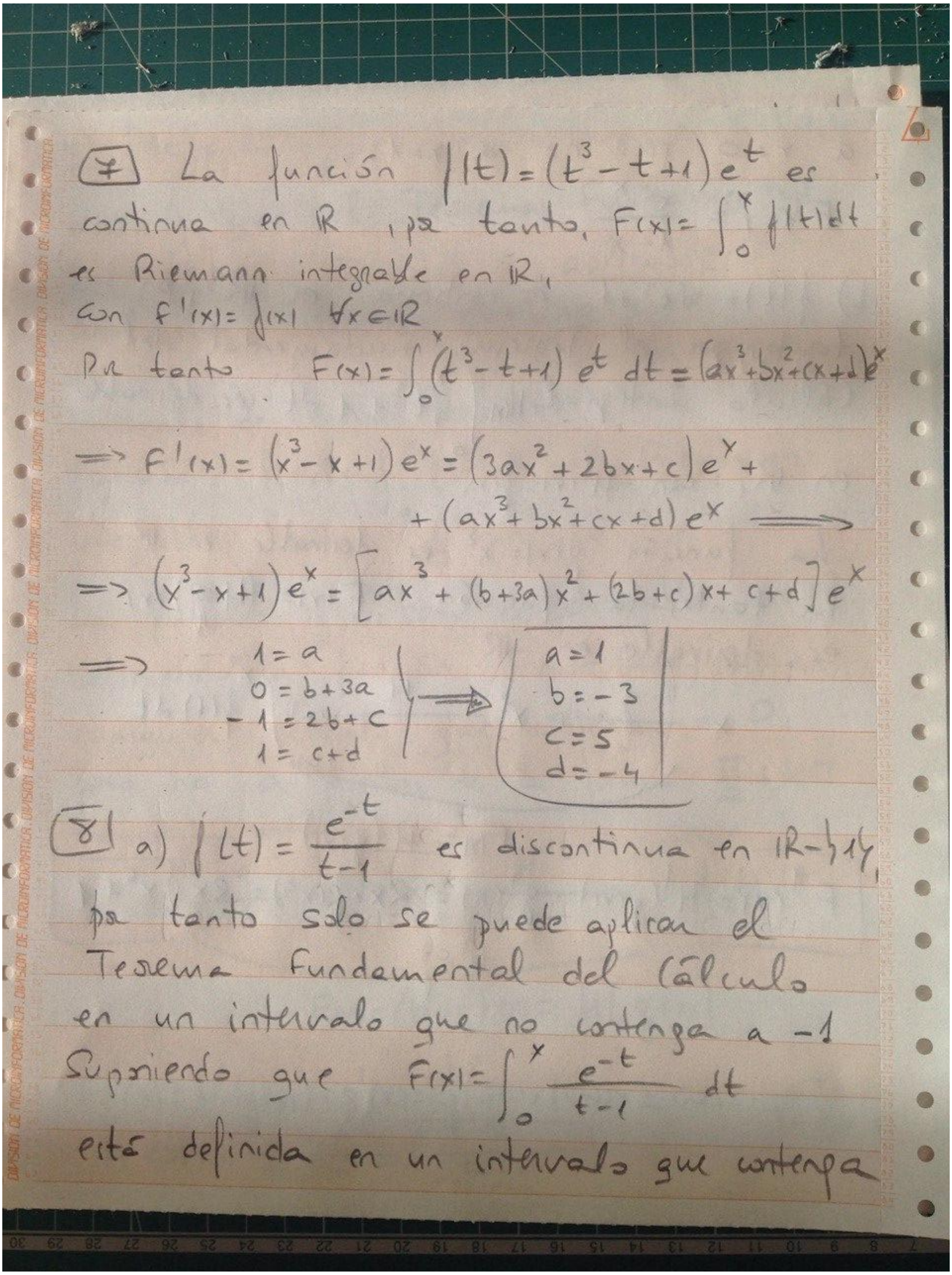
$$\lim_{x\to 0^+} \frac{F(x)}{\text{sen}^3(x)} = \frac{\int_0^0 f(t)\,dt}{0} = \frac{0}{0} = \text{IND.} \xrightarrow{\text{L'Hosp.}}$$

$$\lim_{x\to 0^+} \frac{F'(x)}{3\,\text{sen}^2 x \cdot \cos x} \underset{\boxed{\cos x \to 1}}{=} \lim_{x\to 0^+} \frac{\text{arctg}(x^2)}{3\,\text{sen}^2 x} = \frac{0}{0} \Longrightarrow$$

$$\xrightarrow{\text{L'Hosp.}} \lim_{x\to 0^+} \frac{\frac{1}{1+x^4}\cdot 2x}{6\,\text{sen}x \cdot \cos x} = \lim_{x\to 0^+} \frac{2x}{6(1+x^4)\,\text{sen}x \cos x} =$$

$$\underset{\substack{1+x^4 \to 1 \\ \cos x \to 1}}{=} \lim_{x\to 0^+} \frac{2x}{6\,\text{sen}x} = \lim_{x\to 0^+} \frac{x}{3\,\text{sen}x} \xrightarrow[\text{L'Hosp.}]{}$$

$$\Longrightarrow \lim_{x\to 0^+} \frac{1}{3\cos x} = \boxed{\frac{1}{3}}$$



$\boxed{7}$ La función $f(t)=(t^3-t+1)e^t$ es continua en $\mathbb{R}$, por tanto, $F(x)=\int_0^x f(t)dt$ es Riemann integrable en $\mathbb{R}$, con $f'(x)=f(x)$ $\forall x\in\mathbb{R}$

Por tanto $F(x)=\int_0^x (t^3-t+1)\,e^t\,dt=(ax^3+bx^2+cx+d)e^x$

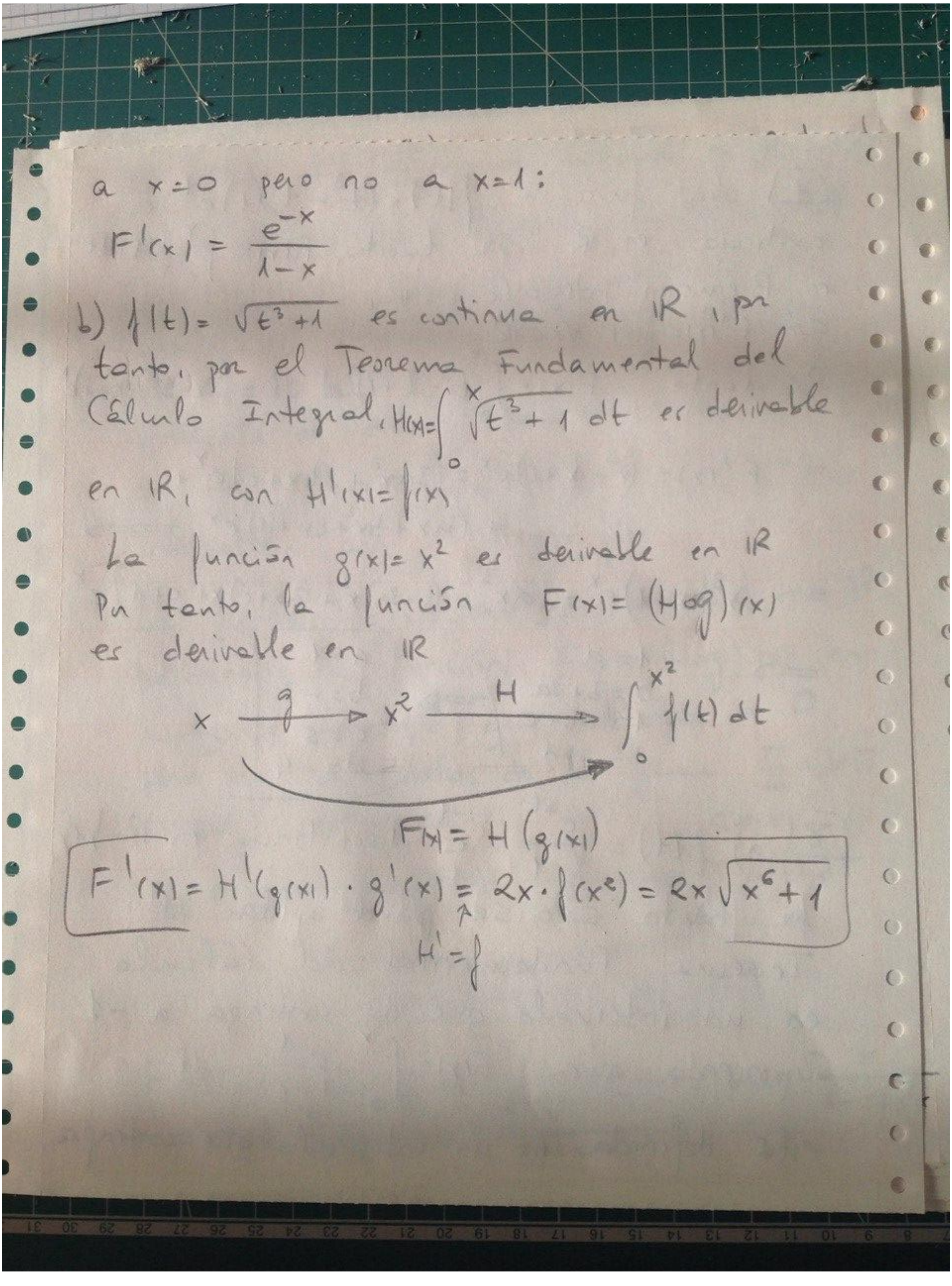
$$\Longrightarrow F'(x)=(x^3-x+1)e^x=(3ax^2+2bx+c)e^x+$$
$$+(ax^3+bx^2+cx+d)e^x \Longrightarrow$$

$$\Longrightarrow (x^3-x+1)e^x=\left[ax^3+(b+3a)x^2+(2b+c)x+c+d\right]e^x$$

$$\Longrightarrow \left.\begin{array}{l} 1=a \\ 0=b+3a \\ -1=2b+c \\ 1=c+d \end{array}\right\} \Longrightarrow \boxed{\begin{array}{l} a=1 \\ b=-3 \\ c=5 \\ d=-4 \end{array}}$$

$\boxed{8}$ a) $f(t)=\dfrac{e^{-t}}{t-1}$ es discontinua en $\mathbb{R}-\{1\}$, por tanto solo se puede aplicar el Teorema Fundamental del Cálculo en un intervalo que no contenga a -1

Suponiendo que $F(x)=\int_0^x \dfrac{e^{-t}}{t-1}\,dt$ esté definida en un intervalo que contenga



a $x=0$ pero no a $x=1$:

$$F'(x) = \frac{e^{-x}}{1-x}$$

b) $f(t) = \sqrt{t^3+1}$ es continua en $\mathbb{R}$, por tanto, por el Teorema Fundamental del Cálculo Integral, $H(x)=\int_0^x \sqrt{t^3+1}\,dt$ es derivable en $\mathbb{R}$, con $H'(x)=f(x)$

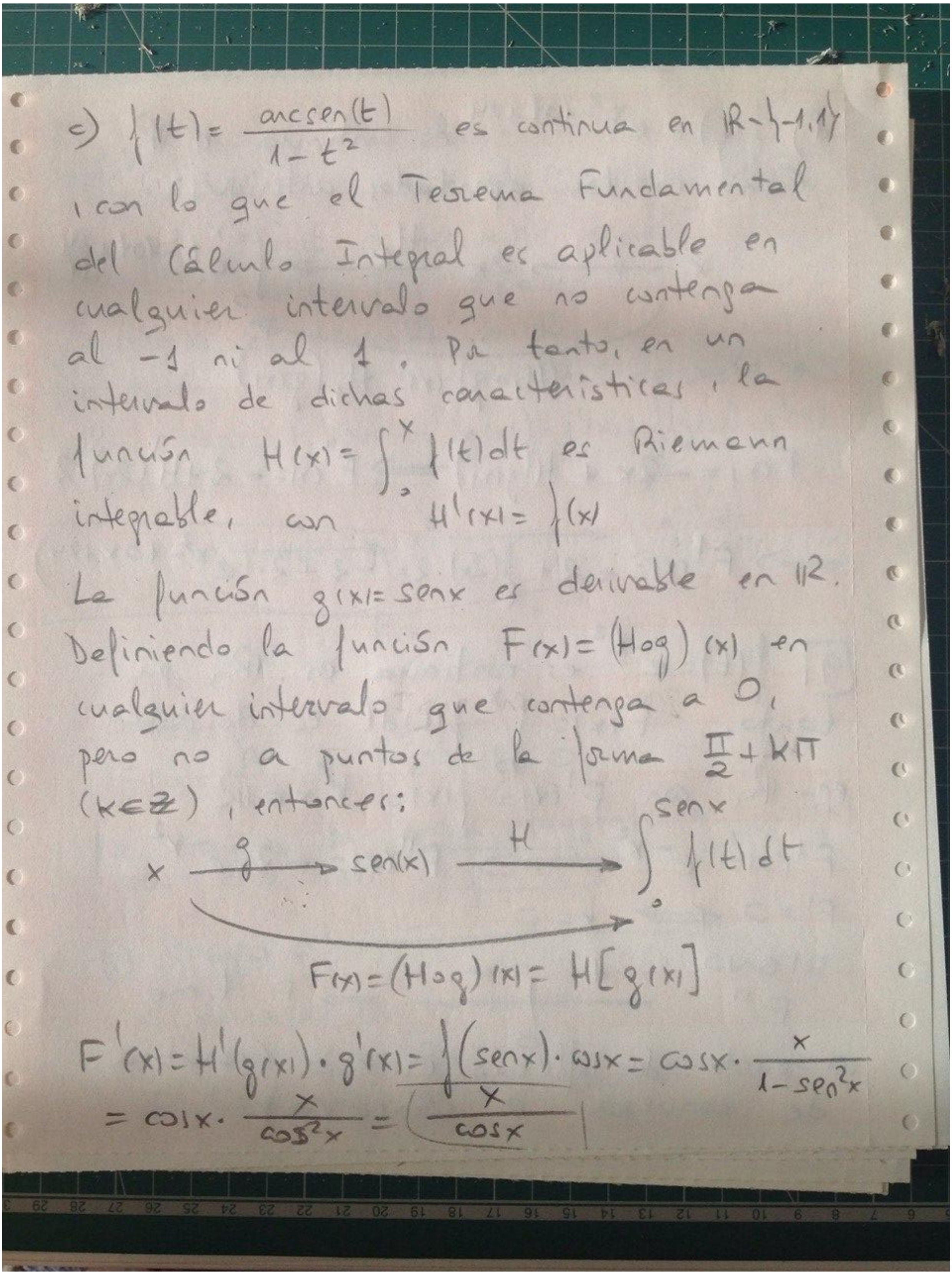
La función $g(x)=x^2$ es derivable en $\mathbb{R}$

Por tanto, la función $F(x)=(H\circ g)(x)$ es derivable en $\mathbb{R}$

$$\begin{tikzcd} x \arrow[r, "g"] \arrow[rr, bend right] & x^2 \arrow[r, "H"] & \int_0^{x^2} f(t)\,dt \end{tikzcd}$$

$$F(x) = H(g(x))$$

$$\boxed{F'(x) = H'(g(x)) \cdot g'(x) \underset{\substack{\uparrow \\ H'=f}}{=} 2x \cdot f(x^2) = 2x\sqrt{x^6+1}}$$



c) $f(t)=\frac{\operatorname{arcsen}(t)}{1-t^2}$ es continua en $\mathbb{R}-\{-1,1\}$, con lo que el Teorema Fundamental del Cálculo Integral es aplicable en cualquier intervalo que no contenga al -1 ni al 1. Por tanto, en un intervalo de dichas características, la función $H(x)=\int_0^x f(t)dt$ es Riemann integrable, con $H'(x)=f(x)$

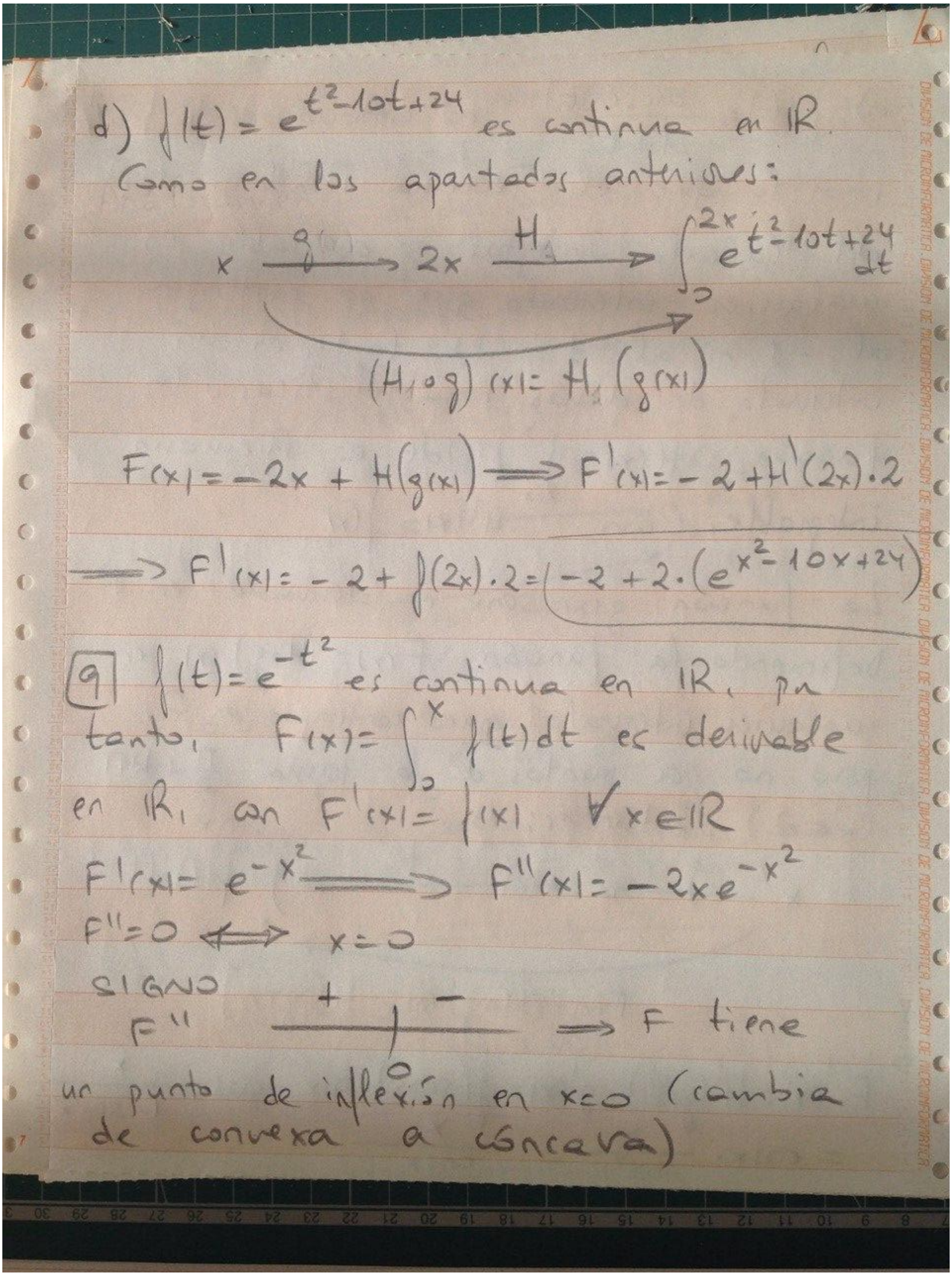
La función $g(x)=\operatorname{sen}x$ es derivable en $\mathbb{R}$. Definiendo la función $F(x)=(H\circ g)(x)$ en cualquier intervalo que contenga a 0, pero no a puntos de la forma $\frac{\pi}{2}+k\pi$ $(k\in\mathbb{Z})$, entonces:

$$\begin{tikzcd} x \arrow[r, "g"] \arrow[rr, bend right] & \operatorname{sen}(x) \arrow[r, "H"] & \int_0^{\operatorname{sen}x} f(t)dt \end{tikzcd}$$

$$F(x)=(H\circ g)(x)=H[g(x)]$$

$$F'(x)=H'(g(x))\cdot g'(x)=f(\operatorname{sen}x)\cdot\cos x=\cos x\cdot\frac{x}{1-\operatorname{sen}^2x}$$

$$=\cos x\cdot\frac{x}{\cos^2x}=\boxed{\frac{x}{\cos x}}$$



d) $f(t) = e^{t^2-10t+24}$ es continua en $\mathbb{R}$.

Como en los apartados anteriores:

$$\begin{tikzcd} x \arrow[r, "g(x)"] \arrow[rr, bend right] & 2x \arrow[r, "H"] & \int_0^{2x} e^{t^2-10t+24}\,dt \end{tikzcd}$$

$(H_1\circ g)(x) = H_1(g(x))$

$F(x) = -2x + H(g(x)) \Longrightarrow F'(x) = -2 + H'(2x)\cdot 2$

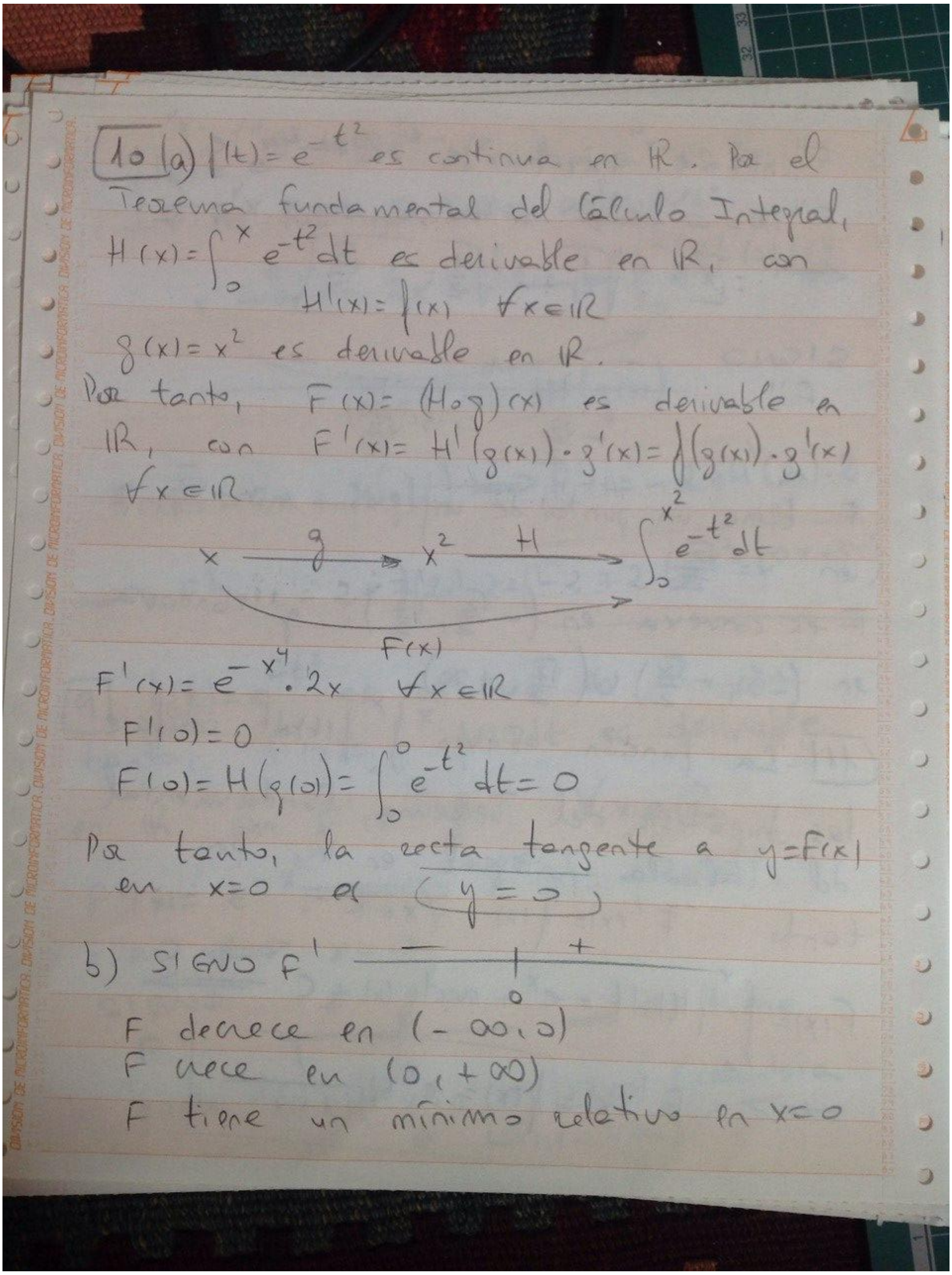
$\Longrightarrow F'(x) = -2 + f(2x)\cdot 2 = \boxed{-2 + 2\cdot(e^{x^2-10x+24})}$

$\boxed{9}$ $f(t) = e^{-t^2}$ es continua en $\mathbb{R}$, por tanto, $F(x) = \int_0^x f(t)\,dt$ es derivable en $\mathbb{R}$, con $F'(x) = f(x)\ \forall x \in \mathbb{R}$

$F'(x) = e^{-x^2} \Longrightarrow F''(x) = -2xe^{-x^2}$

$F'' = 0 \Longleftrightarrow x = 0$

SIGNO F'' $\quad \underset{0}{+ \;|\; -} \Longrightarrow$ F tiene un punto de inflexión en $x=0$ (cambia de convexa a cóncava)



$\boxed{10}$ a) $f(t)=e^{-t^2}$ es continua en $\mathbb{R}$. Por el Teorema fundamental del Cálculo Integral, $H(x)=\int_0^x e^{-t^2}dt$ es derivable en $\mathbb{R}$, con $H'(x)=f(x)\ \forall x\in\mathbb{R}$

$g(x)=x^2$ es derivable en $\mathbb{R}$.

Por tanto, $F(x)=(H\circ g)(x)$ es derivable en $\mathbb{R}$, con $F'(x)=H'(g(x))\cdot g'(x)=f(g(x))\cdot g'(x)$ $\forall x\in\mathbb{R}$

$$\begin{tikzcd} x \arrow[r, "g"] \arrow[rr, bend right, "F(x)"'] & x^2 \arrow[r, "H"] & \int_0^{x^2} e^{-t^2}dt \end{tikzcd}$$

$F'(x)=e^{-x^4}\cdot 2x \quad \forall x\in\mathbb{R}$

$F'(0)=0$

$F(0)=H(g(0))=\int_0^0 e^{-t^2}dt=0$

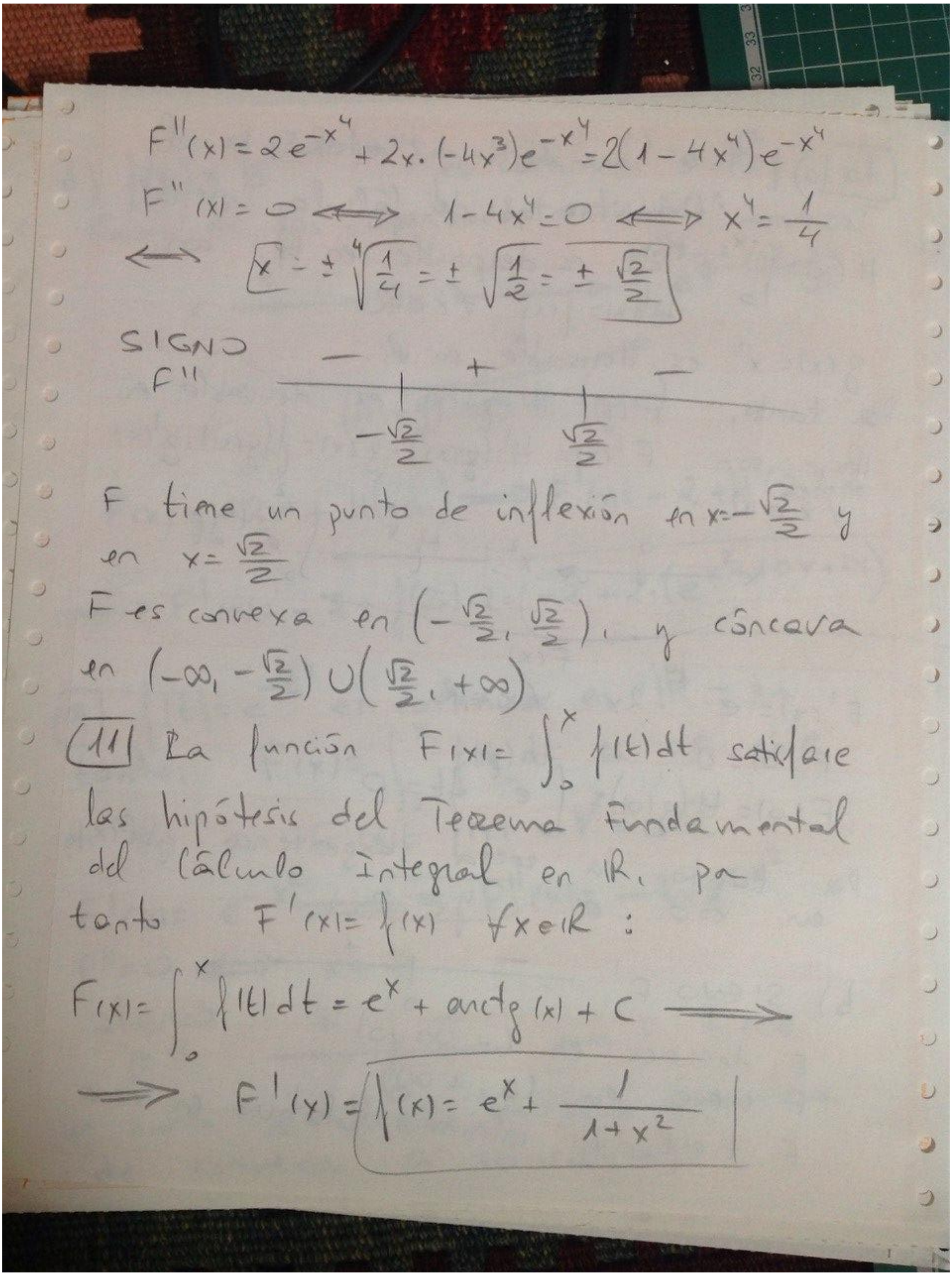
Por tanto, la recta tangente a $y=F(x)$ en $x=0$ es $y=0$

b) SIGNO F' $\quad - \quad \underset{0}{|} \quad +$

F decrece en $(-\infty,0)$

F crece en $(0,+\infty)$

F tiene un mínimo relativo en $x=0$



$$F''(x) = 2e^{-x^4} + 2x\cdot(-4x^3)e^{-x^4} = 2(1-4x^4)e^{-x^4}$$

$$F''(x) = 0 \Longleftrightarrow 1-4x^4 = 0 \Longleftrightarrow x^4 = \frac{1}{4}$$

$$\Longleftrightarrow \boxed{x = \pm\sqrt[4]{\frac{1}{4}} = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}}$$

SIGNO F''

$-$ | $+$ | $-$

$-\frac{\sqrt{2}}{2}$ $\quad$ $\frac{\sqrt{2}}{2}$

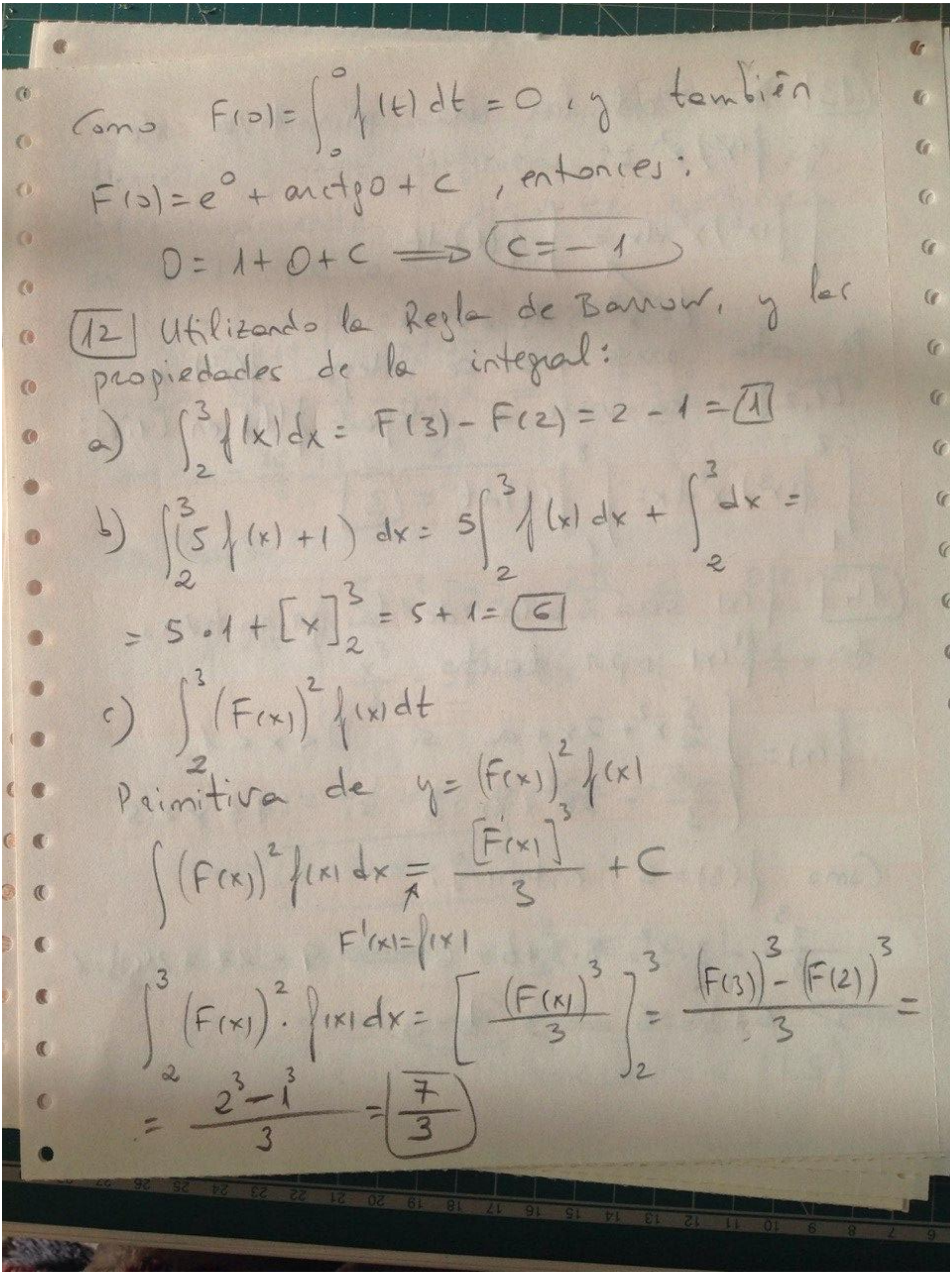
F tiene un punto de inflexión en $x = -\frac{\sqrt{2}}{2}$ y en $x = \frac{\sqrt{2}}{2}$

F es convexa en $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, y cóncava en $\left(-\infty, -\frac{\sqrt{2}}{2}\right) \cup \left(\frac{\sqrt{2}}{2}, +\infty\right)$

$\boxed{11}$ La función $F(x) = \int_0^x f(t)\,dt$ satisface las hipótesis del Teorema Fundamental del cálculo integral en $\mathbb{R}$, por tanto $F'(x) = f(x)\ \forall x \in \mathbb{R}$:

$$F(x) = \int_0^x f(t)\,dt = e^x + \operatorname{arctg}(x) + C \Longrightarrow$$

$$\Longrightarrow F'(y) = \boxed{f(x) = e^x + \frac{1}{1+x^2}}$$



Como $F(0)=\int_0^0 f(t)\,dt = 0$, y también

$F(0)=e^0+\operatorname{arctg} 0+C$, entonces:

$$0 = 1+0+C \Longrightarrow \boxed{C=-1}$$

$\boxed{12}$ Utilizando la Regla de Barrow, y las propiedades de la integral:

a) $\int_2^3 f(x)\,dx = F(3)-F(2) = 2-1=\boxed{1}$

b) $\int_2^3 (5f(x)+1)\,dx = 5\int_2^3 f(x)\,dx + \int_2^3 dx =$

$= 5\cdot 1 + [x]_2^3 = 5+1=\boxed{6}$

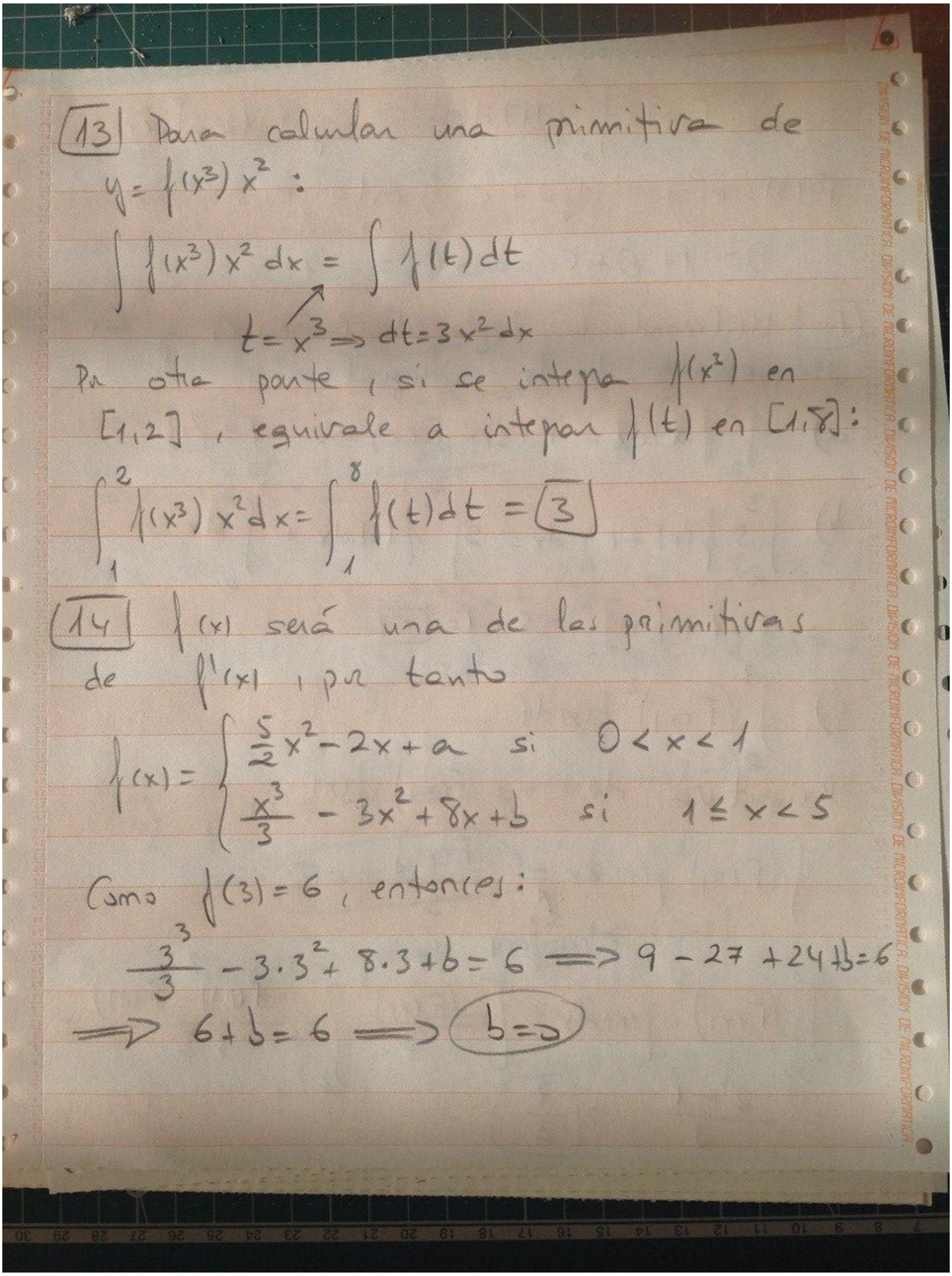
c) $\int_2^3 (F(x))^2 f(x)\,dt$

Primitiva de $y=(F(x))^2 f(x)$

$$\int (F(x))^2 f(x)\,dx \underset{\substack{\uparrow \\ F'(x)=f(x)}}{=} \frac{[F'(x)]^3}{3}+C$$

$$\int_2^3 (F(x))^2\cdot f(x)\,dx = \left[\frac{(F(x))^3}{3}\right]_2^3 = \frac{(F(3))^3-(F(2))^3}{3} =$$

$$= \frac{2^3-1^3}{3} = \boxed{\frac{7}{3}}$$



$\boxed{13}$ Para calcular una primitiva de $y = f(x^3)\,x^2$:

$$\int f(x^3)\,x^2\,dx = \int f(t)\,dt$$

$$t = x^3 \Longrightarrow dt = 3x^2\,dx$$

Por otra parte, si se integra $f(x^3)$ en $[1,2]$, equivale a integrar $f(t)$ en $[1,8]$:

$$\int_1^2 f(x^3)\,x^2\,dx = \int_1^8 f(t)\,dt = \boxed{3}$$

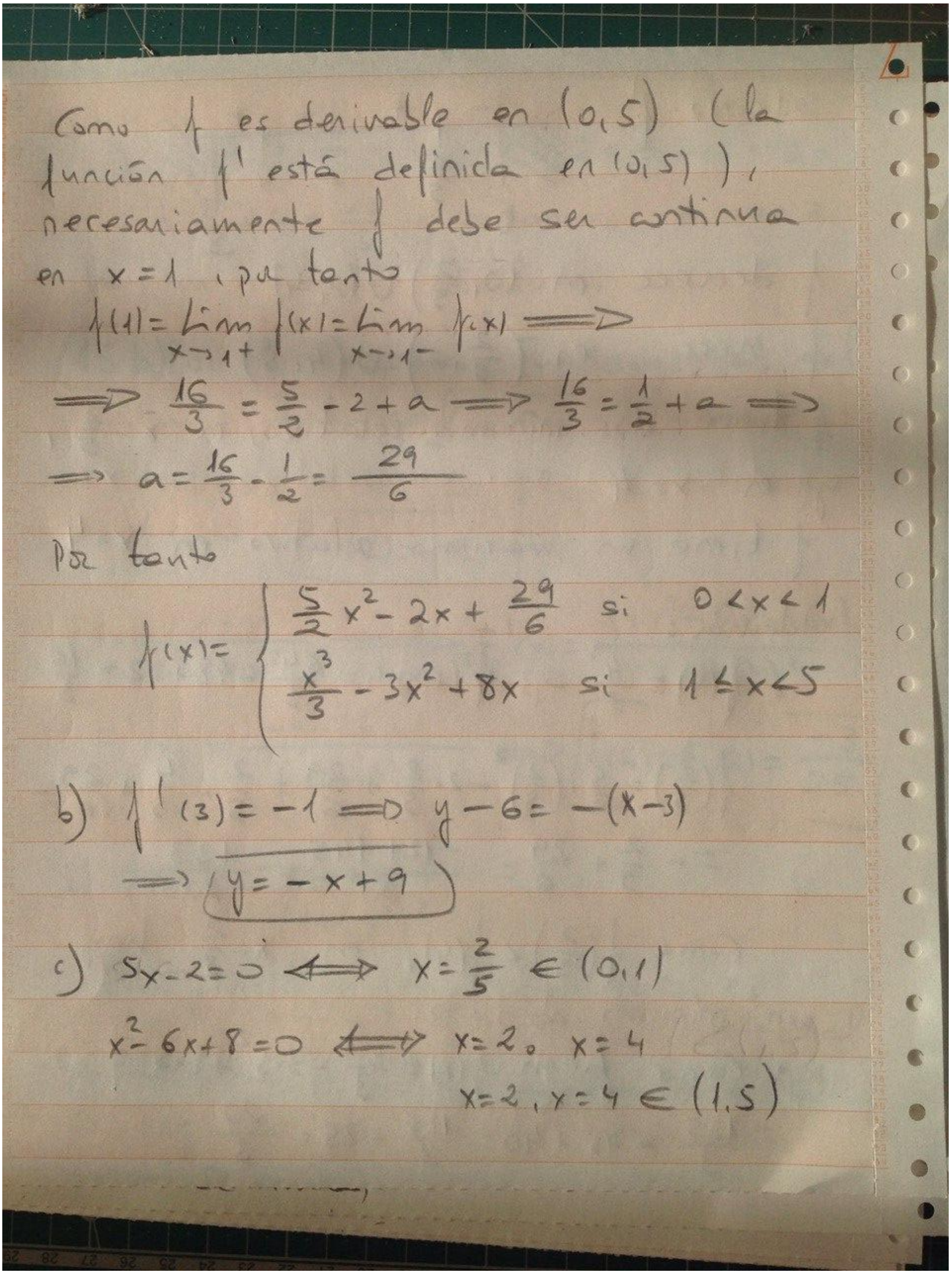
$\boxed{14}$ $f(x)$ será una de las primitivas de $f'(x)$, por tanto

$$f(x) = \begin{cases} \frac{5}{2}x^2 - 2x + a & \text{si} \quad 0 < x < 1 \\ \frac{x^3}{3} - 3x^2 + 8x + b & \text{si} \quad 1 \leq x < 5 \end{cases}$$

Como $f(3) = 6$, entonces:

$$\frac{3^3}{3} - 3\cdot 3^2 + 8\cdot 3 + b = 6 \Longrightarrow 9 - 27 + 24 + b = 6$$

$$\Longrightarrow 6 + b = 6 \Longrightarrow \boxed{b = 0}$$



Como f es derivable en (0,5) (la función f' está definida en (0,5)), necesariamente f debe ser continua en x = 1, por tanto

$$f(1)=\lim_{x\to 1^+} f(x)=\lim_{x\to 1^-} f(x) \Longrightarrow$$

$$\Longrightarrow \frac{16}{3}=\frac{5}{2}-2+a \Longrightarrow \frac{16}{3}=\frac{1}{2}+a \Longrightarrow$$

$$\Longrightarrow a=\frac{16}{3}-\frac{1}{2}=\frac{29}{6}$$

Por tanto

$$f(x)=\begin{cases} \frac{5}{2}x^2-2x+\frac{29}{6} & \text{si } 0<x<1 \\ \frac{x^3}{3}-3x^2+8x & \text{si } 1\le x<5 \end{cases}$$

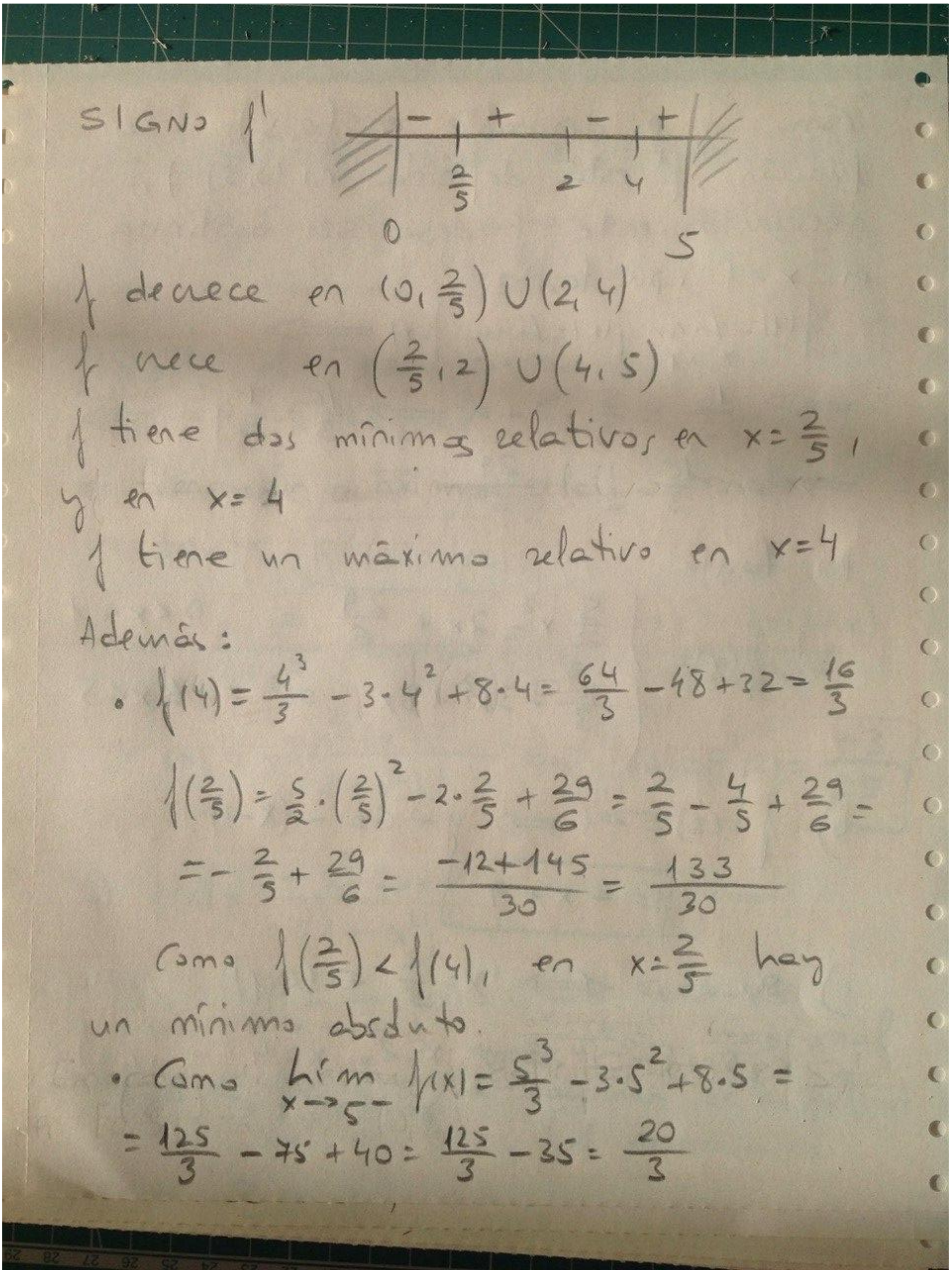
b) $f'(3)=-1 \Longrightarrow y-6=-(x-3)$

$\Longrightarrow \boxed{y=-x+9}$

c) $5x-2=0 \Longleftrightarrow x=\frac{2}{5}\in(0,1)$

$x^2-6x+8=0 \Longleftrightarrow x=2$ o $x=4$

$x=2,\ x=4\in(1,5)$



SIGNO f'

$-\quad +\quad -\quad +$

$0\quad \frac{2}{5}\quad 2\quad 4\quad 5$

f decrece en $(0, \frac{2}{5}) \cup (2, 4)$

f crece en $(\frac{2}{5}, 2) \cup (4, 5)$

f tiene dos mínimos relativos en $x = \frac{2}{5}$, y en $x = 4$

f tiene un máximo relativo en $x = 4$

Además:

- $f(4) = \frac{4^3}{3} - 3 \cdot 4^2 + 8 \cdot 4 = \frac{64}{3} - 48 + 32 = \frac{16}{3}$

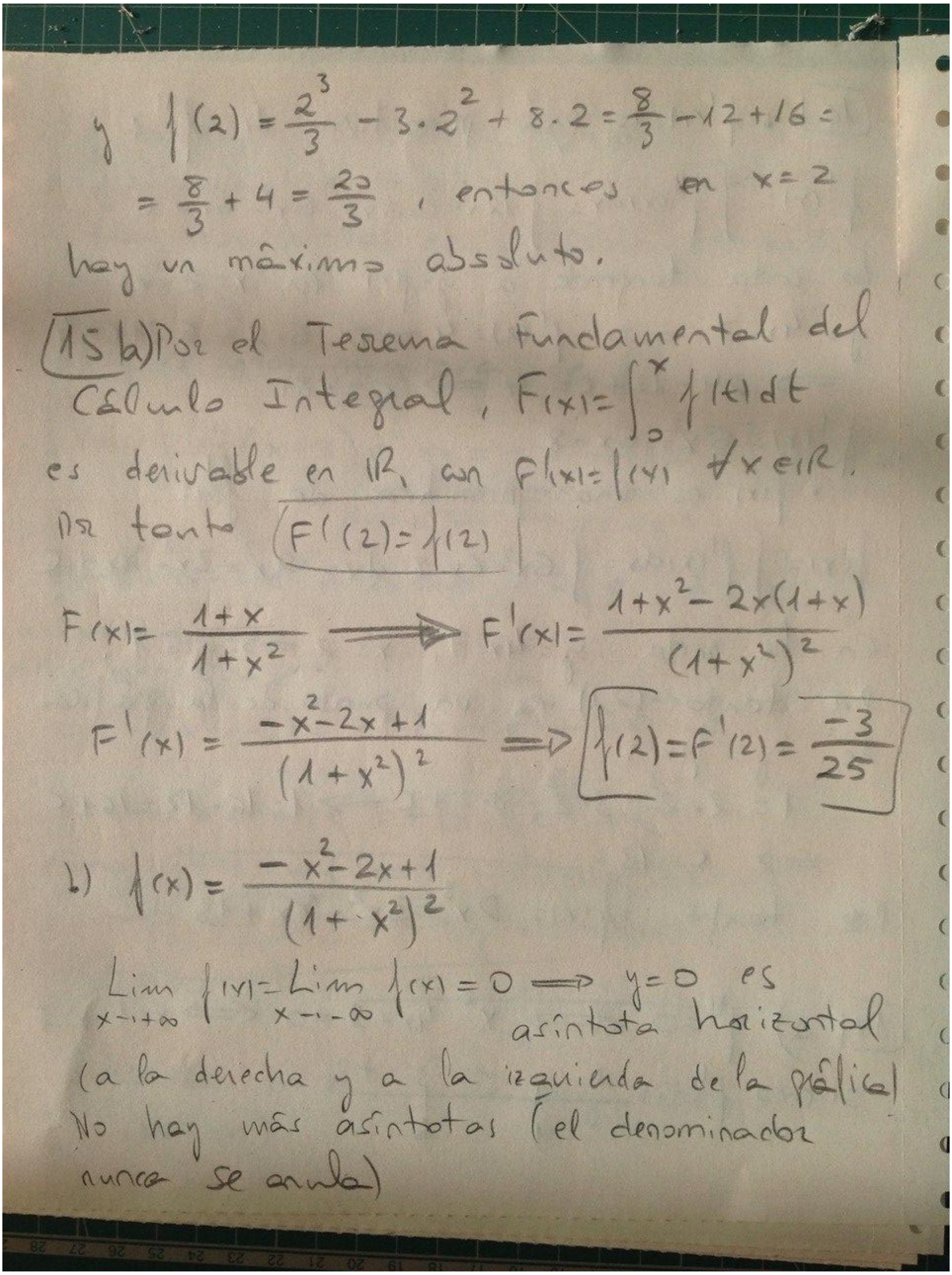
$$f\left(\frac{2}{5}\right) = \frac{5}{2} \cdot \left(\frac{2}{5}\right)^2 - 2 \cdot \frac{2}{5} + \frac{29}{6} = \frac{2}{5} - \frac{4}{5} + \frac{29}{6} =$$

$$= -\frac{2}{5} + \frac{29}{6} = \frac{-12+145}{30} = \frac{133}{30}$$

Como $f(\frac{2}{5}) < f(4)$, en $x = \frac{2}{5}$ hay un mínimo absoluto.

- Como $\lim_{x \to 5^-} f(x) = \frac{5^3}{3} - 3 \cdot 5^2 + 8 \cdot 5 =$

$$= \frac{125}{3} - 75 + 40 = \frac{125}{3} - 35 = \frac{20}{3}$$



y $f(2) = \frac{2^3}{3} - 3\cdot 2^2 + 8\cdot 2 = \frac{8}{3} - 12 + 16 =$

$= \frac{8}{3} + 4 = \frac{20}{3}$, entonces en $x = 2$ hay un máximo absoluto.

(15 a) Por el Teorema Fundamental del Cálculo Integral, $F(x) = \int_0^x f(t)\,dt$ es derivable en $\mathbb{R}$, con $F'(x) = f(x)\ \forall x \in \mathbb{R}$.

Por tanto $\boxed{F'(2) = f(2)}$

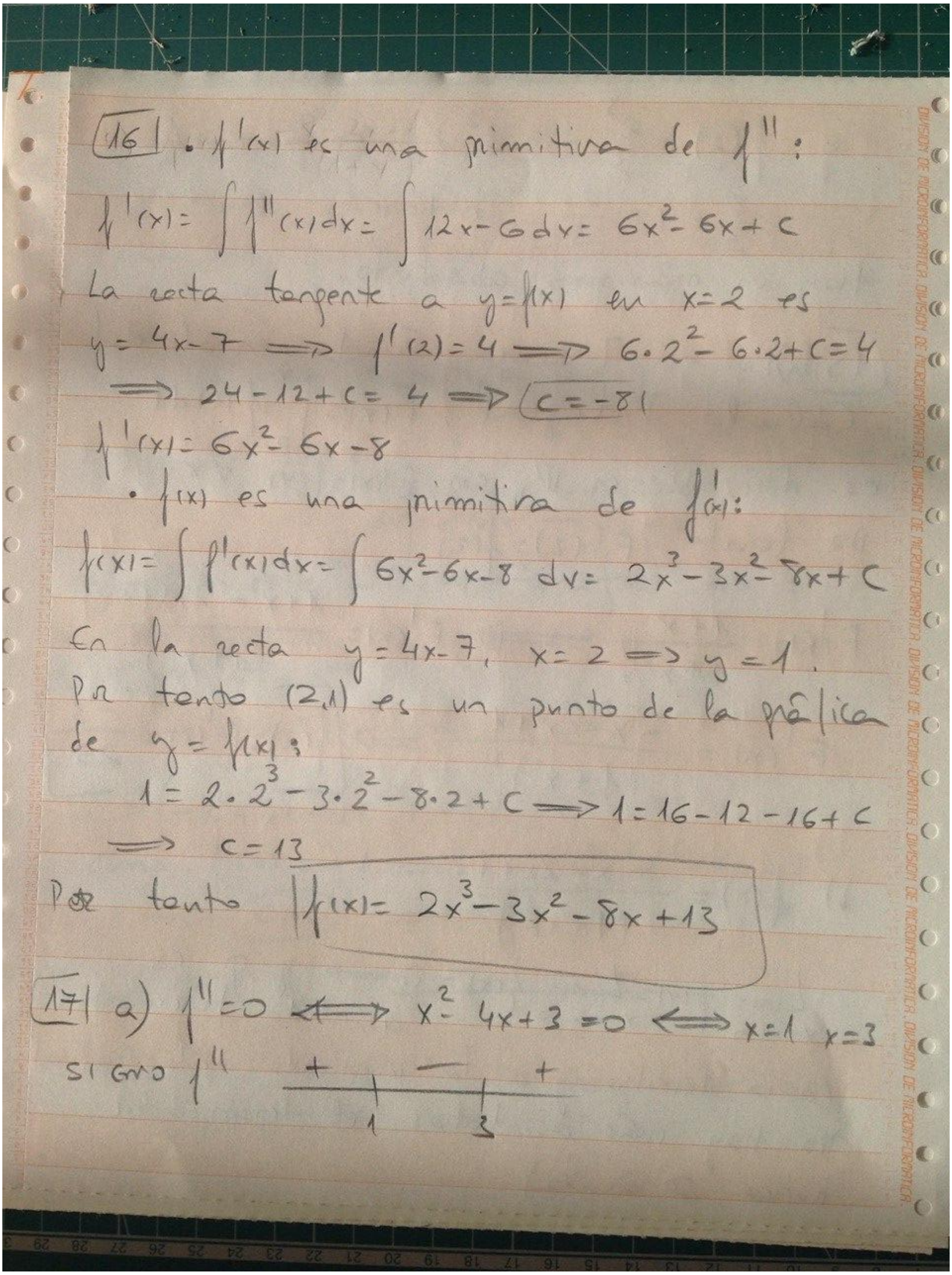
$$F(x) = \frac{1+x}{1+x^2} \Longrightarrow F'(x) = \frac{1+x^2 - 2x(1+x)}{(1+x^2)^2}$$

$$F'(x) = \frac{-x^2-2x+1}{(1+x^2)^2} \Longrightarrow \boxed{f(2) = F'(2) = \frac{-3}{25}}$$

b) $f(x) = \frac{-x^2-2x+1}{(1+x^2)^2}$

$\lim_{x\to+\infty} f(x) = \lim_{x\to-\infty} f(x) = 0 \Longrightarrow y = 0$ es asíntota horizontal (a la derecha y a la izquierda de la gráfica)

No hay más asíntotas (el denominador nunca se anula)



$\boxed{16}$ • $f'(x)$ es una primitiva de f'':

$$f'(x) = \int f''(x)\,dx = \int 12x - 6\,dx = 6x^2 - 6x + C$$

La recta tangente a $y=f(x)$ en $x=2$ es $y = 4x-7 \Longrightarrow f'(2) = 4 \Longrightarrow 6\cdot 2^2 - 6\cdot 2 + C = 4$

$\Longrightarrow 24 - 12 + C = 4 \Longrightarrow \boxed{C = -8}$

$f'(x) = 6x^2 - 6x - 8$

• $f(x)$ es una primitiva de $f'(x)$:

$$f(x) = \int f'(x)\,dx = \int 6x^2 - 6x - 8\,dx = 2x^3 - 3x^2 - 8x + C$$

En la recta $y = 4x-7$, $x = 2 \Longrightarrow y = 1$.
Por tanto $(2,1)$ es un punto de la gráfica de $y = f(x)$:

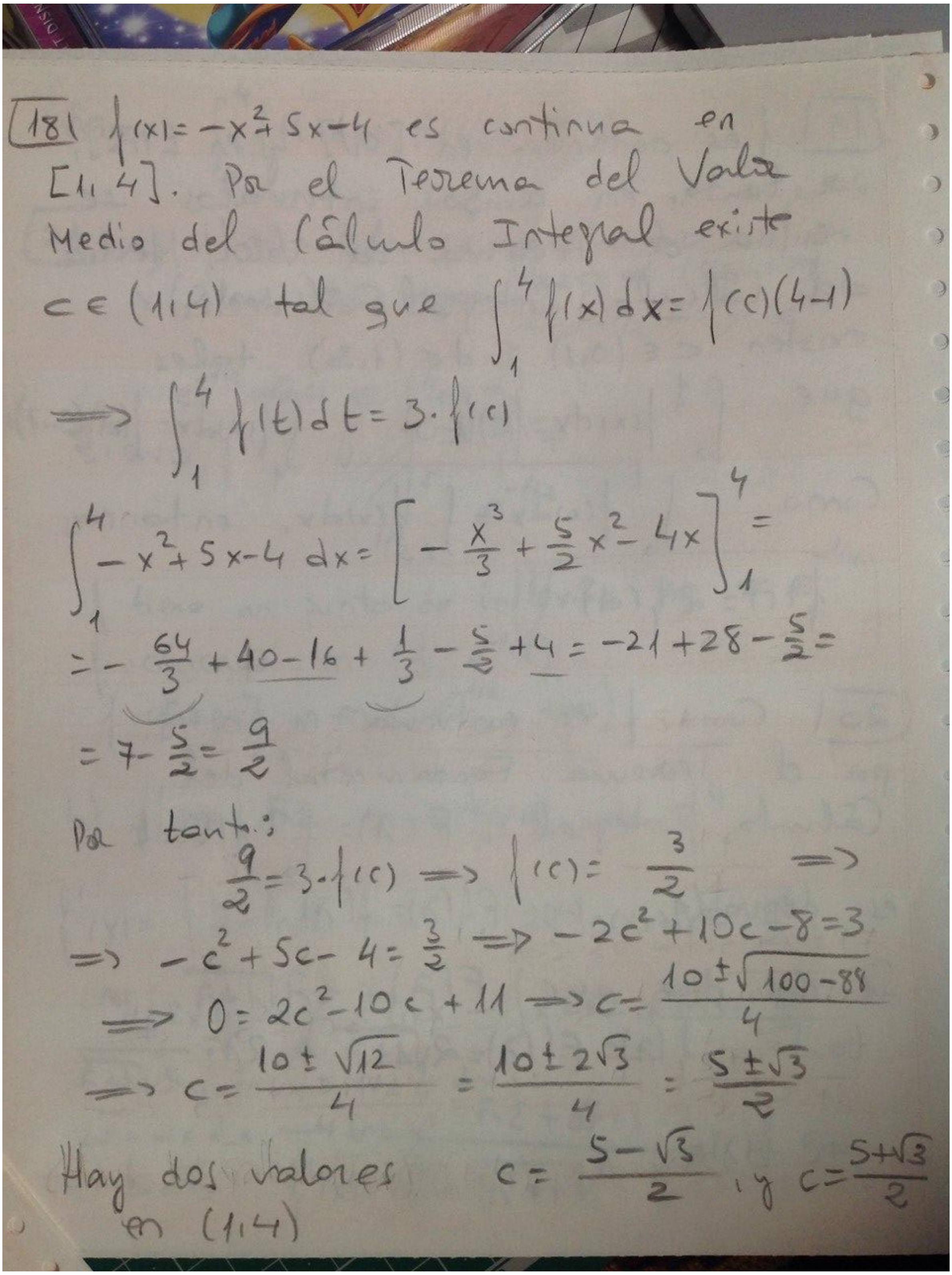
$1 = 2\cdot 2^3 - 3\cdot 2^2 - 8\cdot 2 + C \Longrightarrow 1 = 16 - 12 - 16 + C$

$\Longrightarrow C = 13$

Por tanto $\boxed{f(x) = 2x^3 - 3x^2 - 8x + 13}$

$\boxed{17}$ a) $f'' = 0 \Longleftrightarrow x^2 - 4x + 3 = 0 \Longleftrightarrow x=1 \quad x=3$

signo f''



$\boxed{18}$ $f(x) = -x^2+5x-4$ es continua en $[1,4]$. Por el Teorema del Valor Medio del Cálculo Integral existe $c \in (1,4)$ tal que $\int_1^4 f(x)\,dx = f(c)(4-1)$

$$\Longrightarrow \int_1^4 f(t)\,dt = 3\cdot f(c)$$

$$\int_1^4 -x^2+5x-4\,dx = \left[-\frac{x^3}{3}+\frac{5}{2}x^2-4x\right]_1^4 =$$

$$= -\frac{64}{3}+\underline{40-16}+\frac{1}{3}-\frac{5}{2}+\underline{4} = -21+28-\frac{5}{2} =$$

$$= 7-\frac{5}{2} = \frac{9}{2}$$

Por tanto:

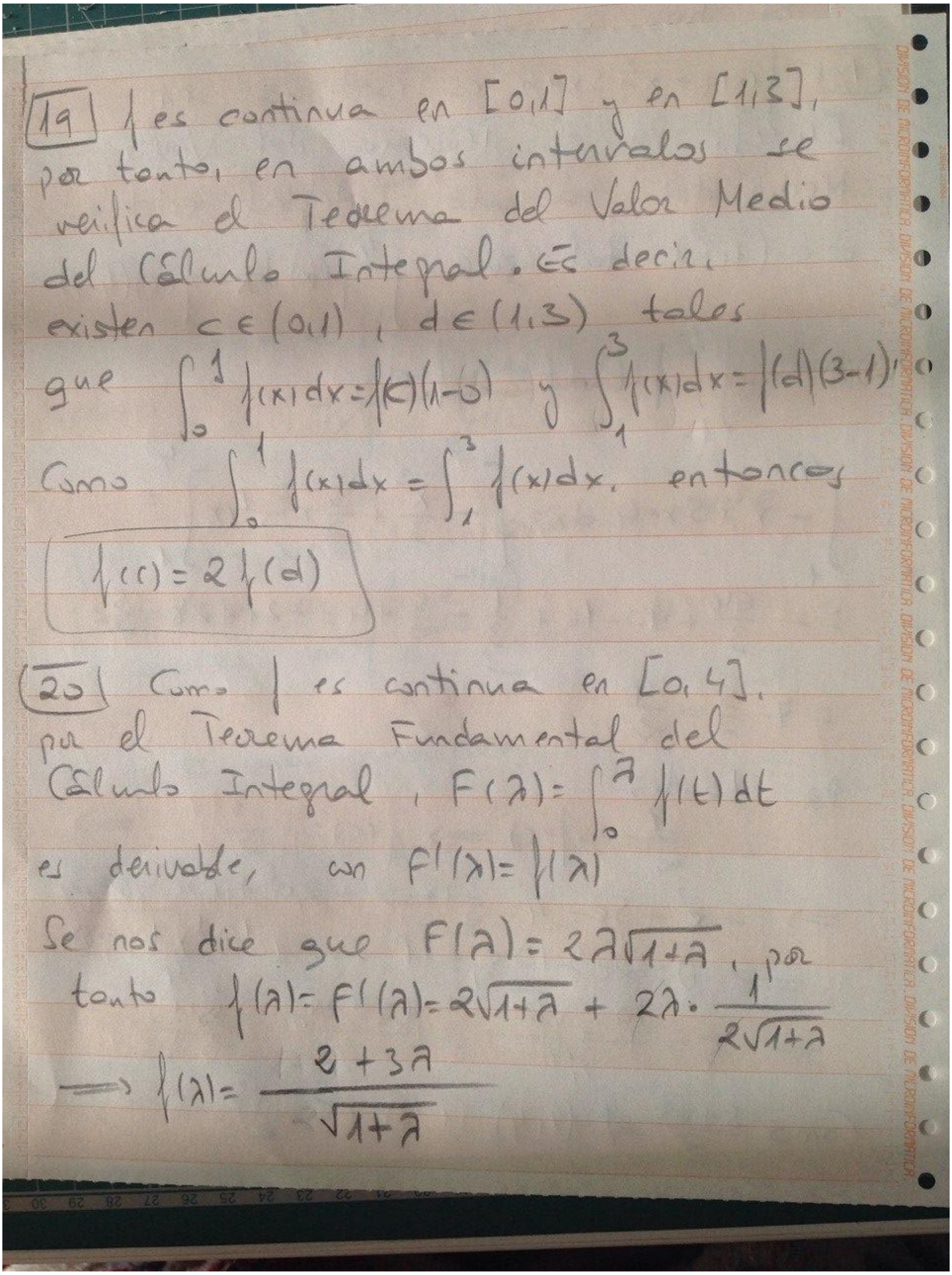
$$\frac{9}{2} = 3\cdot f(c) \Longrightarrow f(c) = \frac{3}{2} \Longrightarrow$$

$$\Longrightarrow -c^2+5c-4 = \frac{3}{2} \Longrightarrow -2c^2+10c-8=3$$

$$\Longrightarrow 0 = 2c^2-10c+11 \Longrightarrow c = \frac{10\pm\sqrt{100-88}}{4}$$

$$\Longrightarrow c = \frac{10\pm\sqrt{12}}{4} = \frac{10\pm 2\sqrt{3}}{4} = \frac{5\pm\sqrt{3}}{2}$$

Hay dos valores $c = \frac{5-\sqrt{3}}{2}$ y $c = \frac{5+\sqrt{3}}{2}$ en $(1,4)$



19 f es continua en $[0,1]$ y en $[1,3]$, por tanto, en ambos intervalos se verifica el Teorema del Valor Medio del Cálculo Integral. Es decir, existen $c \in (0,1)$, $d \in (1,3)$ tales que $\int_0^1 f(x)dx = f(c)(1-0)$ y $\int_1^3 f(x)dx = f(d)(3-1)$.

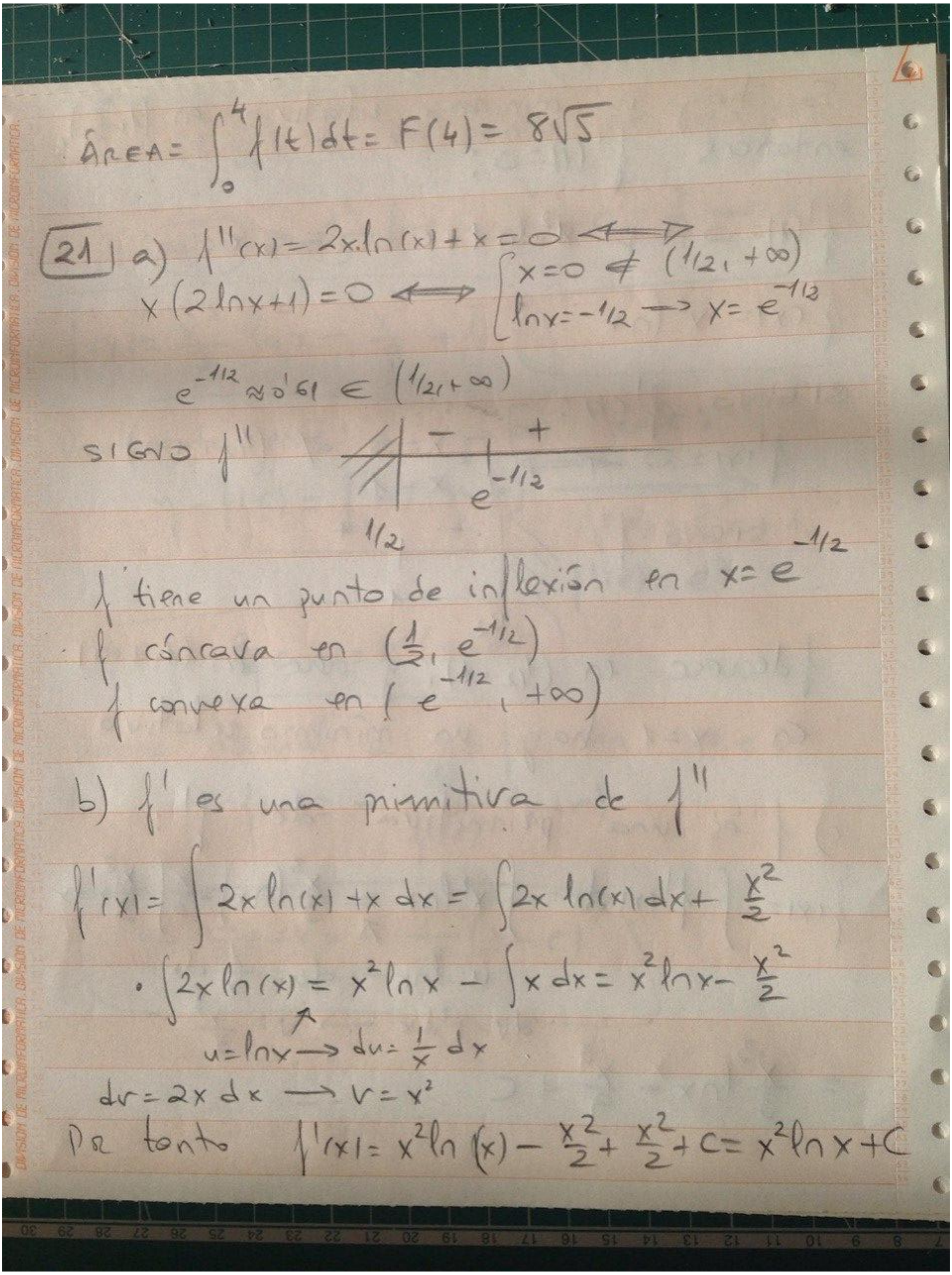
Como $\int_0^1 f(x)dx = \int_1^3 f(x)dx$, entonces

$$\boxed{f(c) = 2f(d)}$$

20 Como f es continua en $[0,4]$, por el Teorema Fundamental del Cálculo Integral, $F(\lambda) = \int_0^\lambda f(t)dt$ es derivable, con $F'(\lambda) = f(\lambda)$

Se nos dice que $F(\lambda) = 2\lambda\sqrt{1+\lambda}$, por tanto $f(\lambda) = F'(\lambda) = 2\sqrt{1+\lambda} + 2\lambda \cdot \frac{1}{2\sqrt{1+\lambda}}$

$$\Longrightarrow f(\lambda) = \frac{2+3\lambda}{\sqrt{1+\lambda}}$$



ÁREA $= \int_0^4 f(t)\,dt = F(4) = 8\sqrt{5}$

21 a) $f''(x) = 2x\ln(x) + x = 0 \Longleftrightarrow$

$x(2\ln x + 1) = 0 \Longleftrightarrow \begin{cases} x = 0 \notin (1/2, +\infty) \\ \ln x = -1/2 \longrightarrow x = e^{-1/2} \end{cases}$

$e^{-1/2} \approx 0'61 \in (1/2, +\infty)$

SIGNO f''

$-$ $+$

$1/2$ $e^{-1/2}$

f tiene un punto de inflexión en $x = e^{-1/2}$

f cóncava en $(\frac{1}{2}, e^{-1/2})$

f convexa en $(e^{-1/2}, +\infty)$

b) f' es una primitiva de f''

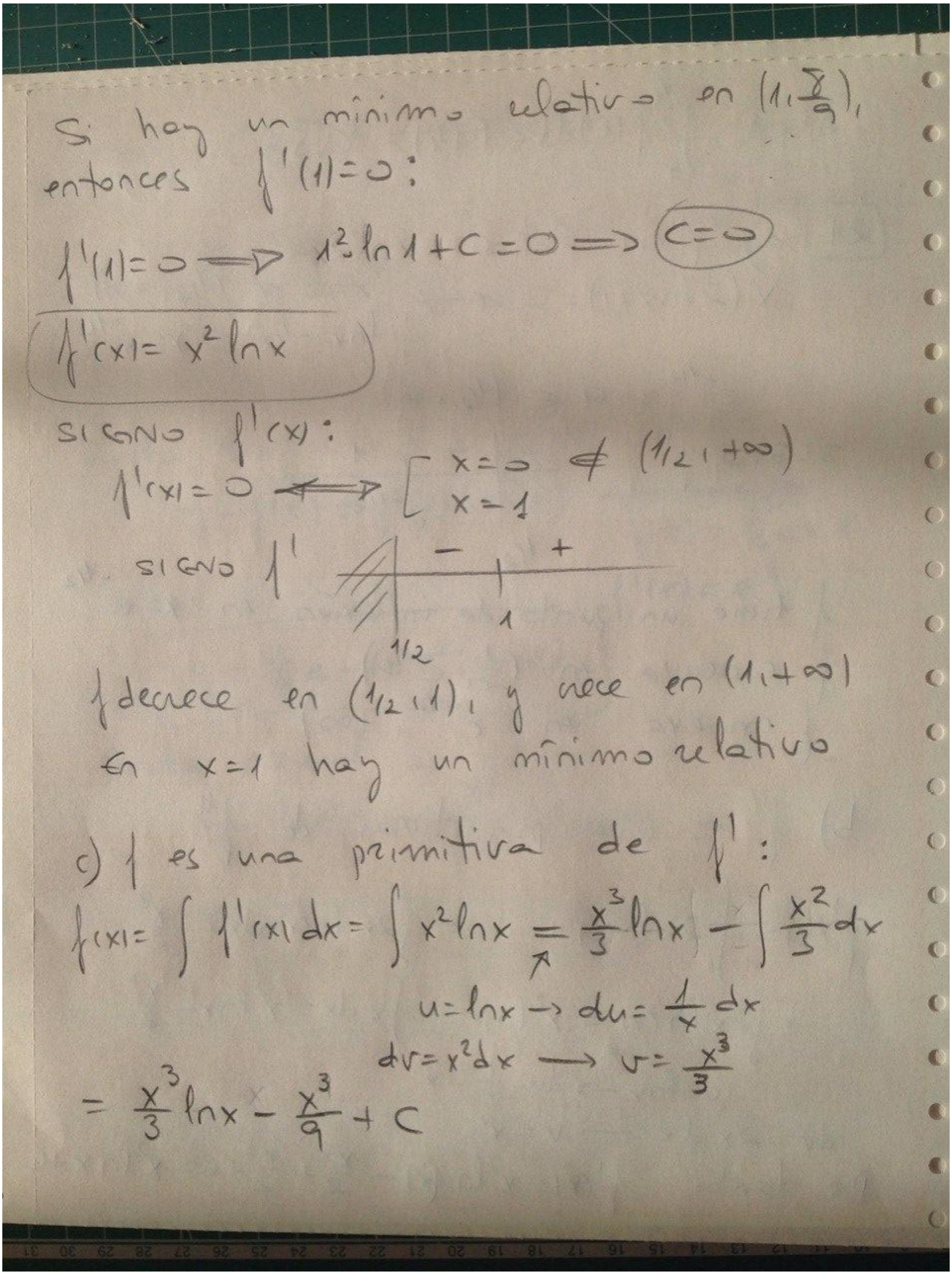
$f'(x) = \int 2x\ln(x) + x\,dx = \int 2x\ln(x)\,dx + \frac{x^2}{2}$

• $\int 2x\ln(x) = x^2\ln x - \int x\,dx = x^2\ln x - \frac{x^2}{2}$

$u = \ln x \longrightarrow du = \frac{1}{x}dx$

$dv = 2x\,dx \longrightarrow v = x^2$

Por tanto $f'(x) = x^2\ln(x) - \frac{x^2}{2} + \frac{x^2}{2} + C = x^2\ln x + C$



Si hay un mínimo relativo en $\left(1, -\frac{8}{9}\right)$, entonces $f'(1)=0$:

$$f'(1)=0 \Longrightarrow 1^2\cdot\ln 1 + C = 0 \Longrightarrow \boxed{C=0}$$

$$\boxed{f'(x)=x^2\ln x}$$

SIGNO $f'(x)$:

$$f'(x)=0 \Longleftrightarrow \left[\begin{array}{l} x=0 \notin (1/2,+\infty) \\ x=1 \end{array}\right.$$

SIGNO f'

f decrece en $(1/2, 1)$, y crece en $(1,+\infty)$

En $x=1$ hay un mínimo relativo

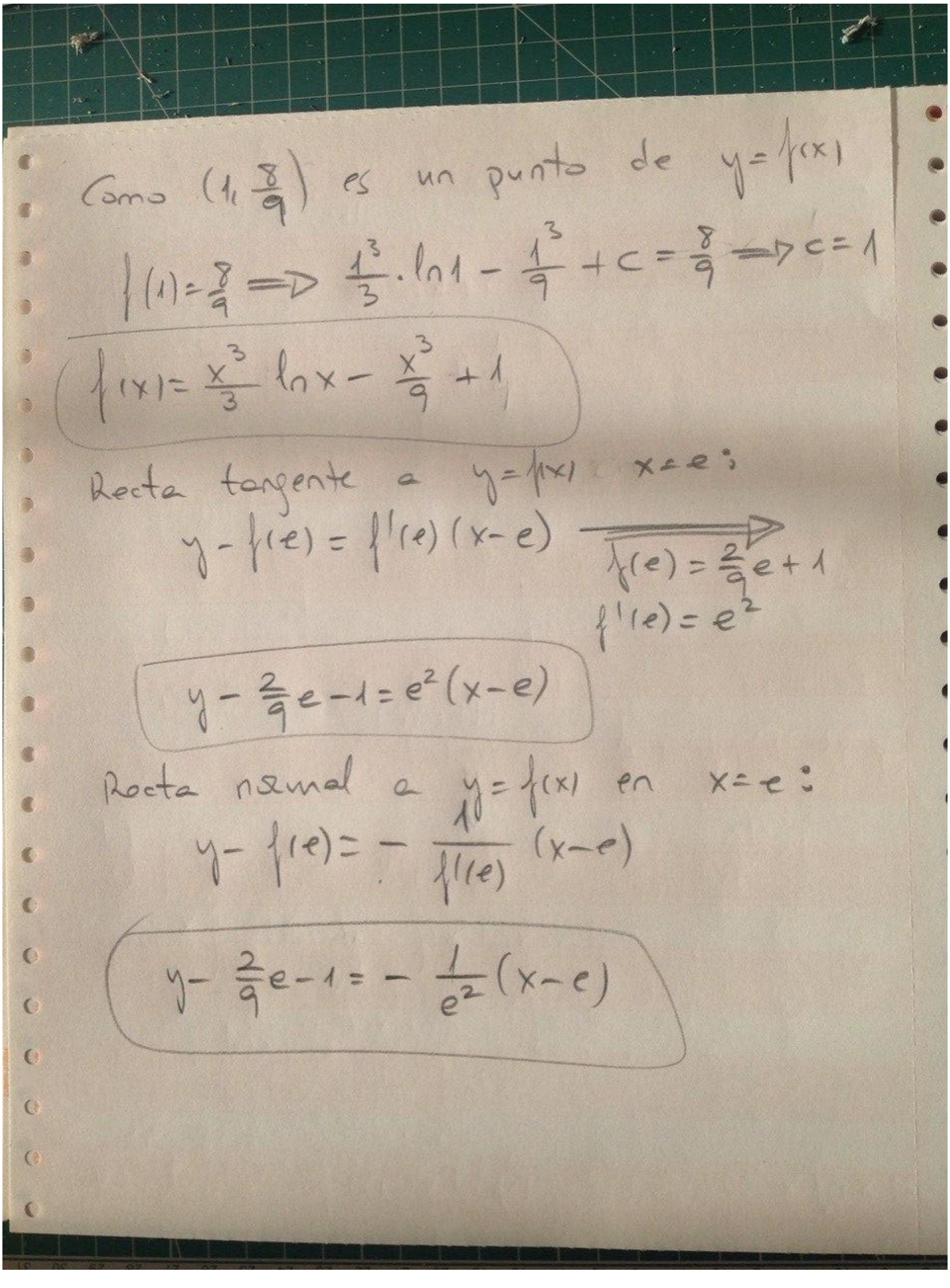
c) f es una primitiva de f':

$$f(x)=\int f'(x)\,dx = \int x^2\ln x \underset{\uparrow}{=} \frac{x^3}{3}\ln x - \int \frac{x^2}{3}\,dx$$

$$u=\ln x \rightarrow du=\frac{1}{x}\,dx$$

$$dv=x^2\,dx \longrightarrow v=\frac{x^3}{3}$$

$$= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$$



Como $(1, \frac{8}{9})$ es un punto de $y=f(x)$

$$f(1)=\frac{8}{9} \Longrightarrow \frac{1^3}{3}\cdot\ln 1-\frac{1^3}{9}+C=\frac{8}{9} \Longrightarrow C=1$$

$$\boxed{f(x)=\frac{x^3}{3}\ln x-\frac{x^3}{9}+1}$$

Recta tangente a $y=f(x)$ $x=e$:

$$y-f(e)=f'(e)(x-e) \Longrightarrow \begin{array}{l} f(e)=\frac{2}{9}e+1 \\ f'(e)=e^2 \end{array}$$

$$\boxed{y-\frac{2}{9}e-1=e^2(x-e)}$$

Recta normal a $y=f(x)$ en $x=e$:

$$y-f(e)=-\frac{1}{f'(e)}(x-e)$$

$$\boxed{y-\frac{2}{9}e-1=-\frac{1}{e^2}(x-e)}$$