Soluciones a los ejercicios de cálculo de áreas

1
58

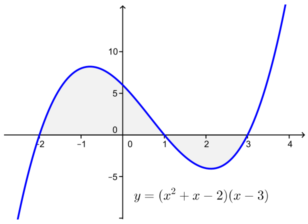
$\left(x^2+x-2\right)\ (x-3)=(x+2)\ (x-1)\ (x-3)$

$\left(x^2+x-2\right)\ (x-3)=x^3-2x^2-5x+6$

$$\text{área}=\left|\int_{-2}^{1}\left(x^3-2x^2-5x+6\right)dx\right|+\left|\int_{1}^{3}\left(x^3-2x^2-5x+6\right)dx\right|=$$

$$=\left|\left[\tfrac{1}{4}x^4-\tfrac{2}{3}x^3-\tfrac{5}{2}x^2+6x\right]_{-2}^{1}\right|+\left|\left[\tfrac{1}{4}x^4-\tfrac{2}{3}x^3-\tfrac{5}{2}x^2+6x\right]_{1}^{3}\right|=$$

$$=\left|\tfrac{37}{12}-\left(-\tfrac{38}{3}\right)\right|+\left|\left(-\tfrac{9}{4}\right)-\tfrac{37}{12}\right|=\tfrac{189}{12}+\tfrac{64}{12}=\boxed{\tfrac{253}{12}\approx 21{,}083\ \text{u}^2}$$

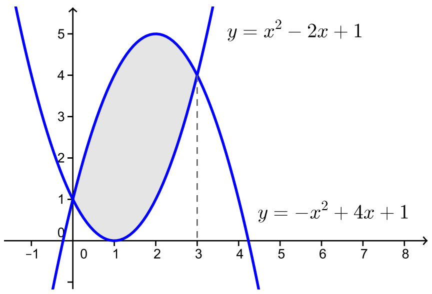


2
59

$\left(x^2-2x+1\right)-\left(-x^2+4x+1\right)=2x^2-6x=2x\,(x-3)$

Raíces del polinomio 0 y 3.

$$\text{Área}=\left|\int_{0}^{3}\left(2x^2-6x\right)dx\right|=\left|\left[\tfrac{2}{3}x^3-3x^2\right]_{0}^{3}\right|=\boxed{9\ \text{u}^2}$$



3
62

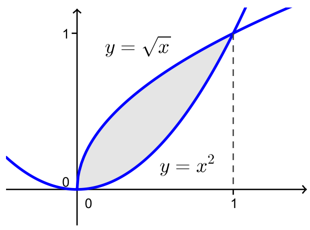
$y=x^2\ ;\ y=\sqrt{x}$

Puntos de corte entre ambas curvas:

$x^2=\sqrt{x}\ ;\ x^4=x\ ;\ x^4-x=0\ ;\ x\left(x^3-1\right)=0$

soluciones: 0 y 1

$$\text{Área}=\left|\int_{0}^{1}\left(\sqrt{x}-x^2\right)dx\right|=\left|\left[\tfrac{2}{3}\sqrt{x^3}-\tfrac{1}{3}x^3\right]_{0}^{1}\right|=\boxed{\tfrac{1}{3}\ \text{u}^2}$$



4
193

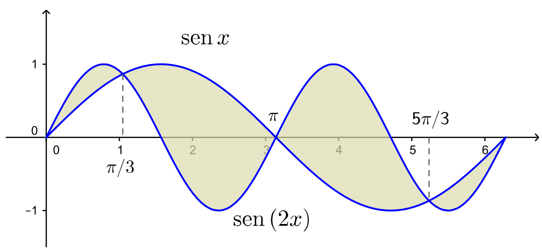
$$\int_{0}^{1/2}\frac{x^3}{\sqrt{1-x^2}}\,dx\ ,\ \left\{\begin{array}{ll}1-x^2=t^2 & ,\ \ t=\sqrt{1-x^2}\\ x^2=1-t^2 & ,\ \ 2x\,dx=-2t\,dt\end{array}\right\}$$

$$\int\frac{x^3}{\sqrt{1-x^2}}\,dx=\int\frac{x^2}{\sqrt{1-x^2}}\,x\,dx=\int\frac{1-t^2}{t}\ (-t)\ dt=\int\ \left(t^2-1\right)\ dt=\tfrac{1}{3}t^3-t=\tfrac{1}{3}\left(\sqrt{1-x^2}\right)^3-\sqrt{1-x^2}$$

$$\int_{0}^{1/2}\frac{x^3}{\sqrt{1-x^2}}\,dx=\left[\tfrac{1}{3}\left(\sqrt{1-x^2}\right)^3-\sqrt{1-x^2}\right]_{0}^{1/2}=\tfrac{1}{3}\left(\sqrt{1-\tfrac{1}{4}}\right)^3-\sqrt{1-\tfrac{1}{4}^2}-\left(\tfrac{1}{3}\left(\sqrt{1}\right)^3-\sqrt{1}\right)=\tfrac{1}{3}\cdot\tfrac{3\sqrt{3}}{8}-\tfrac{\sqrt{3}}{2}-\tfrac{1}{3}+1=$$

$$=\left(\tfrac{1}{8}-\tfrac{1}{2}\right)\sqrt{3}+\tfrac{2}{3}=\boxed{\tfrac{2}{3}-\tfrac{3\sqrt{3}}{8}}$$

5
195



$$\text{Área}=\left|\int_{0}^{\pi/3}\left(\operatorname{sen}(2x)-\operatorname{sen}x\right)\,dx\right|+\left|\int_{\pi/3}^{\pi}\left(\operatorname{sen}(2x)-\operatorname{sen}x\right)\,dx\right|+\left|\int_{\pi}^{5\pi/3}\left(\operatorname{sen}(2x)-\operatorname{sen}x\right)\,dx\right|+\left|\int_{5\pi/3}^{2\pi}\left(\operatorname{sen}(2x)-\operatorname{sen}x\right)\,dx\right|=$$

$$=\left|\left[-\tfrac{1}{2}\cos(2x)+\cos x\right]_{0}^{\pi/3}\right|+\left|\left[-\tfrac{1}{2}\cos(2x)+\cos x\right]_{\pi/3}^{\pi}\right|+\left|\left[-\tfrac{1}{2}\cos(2x)+\cos x\right]_{\pi}^{5\pi/3}\right|+\left|\left[-\tfrac{1}{2}\cos(2x)+\cos x\right]_{5\pi/3}^{2\pi}\right|=$$

$$=\left|-\tfrac{1}{2}\times\left(-\tfrac{1}{2}\right)+\tfrac{1}{2}-\left(-\tfrac{1}{2}+1\right)\right|+\left|-\tfrac{1}{2}-1-\left(\left(-\tfrac{1}{2}\right)\ \left(-\tfrac{1}{2}\right)+\tfrac{1}{2}\right)\right|+\left|-\tfrac{1}{2}\times\left(-\tfrac{1}{2}\right)+\tfrac{1}{2}-\left(-\tfrac{1}{2}-1\right)\right|+\left|-\tfrac{1}{2}+1-\left(\tfrac{1}{4}+\tfrac{1}{2}\right)\right|=$$

$$=\tfrac{1}{4}+\tfrac{9}{4}+\tfrac{9}{4}+\tfrac{1}{4}=\boxed{5\ \text{u}^2}$$

6
4333

$$|2x-1| = \begin{cases} -2x+1 & \text{si } x \le \frac{1}{2} \\ 2x-1 & \text{si } x > \frac{1}{2} \end{cases}$$

$$\int_0^2 |2x-1|\,dx = \int_0^{1/2} (-2x+1)\,dx + \int_{1/2}^2 (2x-1)\,dx = \left[-x^2+x\right]_0^{1/2} + \left[x^2-x\right]_{1/2}^2 =$$

$$= -\frac{1}{4} + \frac{1}{2} + 4 - 2 - \left(\frac{1}{4} - \frac{1}{2}\right) = \frac{1}{4} + 2 + \frac{1}{4} = \boxed{\frac{5}{2}}$$

7
6240

$$\begin{cases} y = -x^2 + ax \\ y = x \end{cases}$$

$$-x^2 + ax = x \ ; \ x^2 - (a-1)\,x = 0 \ ; \ x\,(x - (a-1)) = 0 \ ; \ \begin{cases} x_1 = 0 \\ x_2 = a-1 \end{cases}$$

$$\text{Área } = \left| \int_0^{a-1} \left(-x^2 + ax - x\right)\,dx \right| = \left| \int_0^{a-1} \left(-x^2 + (a-1)\,x\right)\,dx \right| = \left| \left[-\frac{x^3}{3} + \frac{(a-1)x^2}{2} \right]_0^{a-1} \right| = \left| -\frac{(a-1)^3}{3} + \frac{(a-1)(a-1)^2}{2} \right| \underset{a>1}{=} \frac{(a-1)^3}{6}$$

$$\frac{(a-1)^3}{6} = \frac{4}{3} \ ; \ a = 1 + \sqrt[3]{6 \cdot \frac{4}{3}} = 1 + 2 = 3 \ ; \ \boxed{a = 3}$$

8
6413

$$f(x) = |x-1| - 1 = \begin{cases} -x & \text{si } x \le 1 \\ x-2 & \text{si } x > 1 \end{cases} \ ; \ g(x) = \text{sen}\left(\frac{\pi}{2}x\right)$$

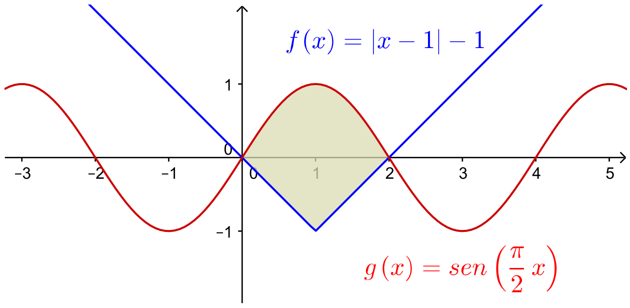
$$x \le 1 : \ -x = \text{sen}\left(\frac{\pi}{2}x\right) \Rightarrow x = 0$$

$$x > 1 : \ x - 2 = \text{sen}\left(\frac{\pi}{2}x\right) \Rightarrow x = 2$$

[La demostración de que 0 y 2 son las únicas soluciones NO es elemental]

$$\text{Área} = \left| \int_0^1 \left(-x - \text{sen}\left(\frac{\pi}{2}x\right)\right)\,dx \right| + \left| \int_1^2 \left(x - 2 - \text{sen}\left(\frac{\pi}{2}x\right)\right)\,dx \right| =$$

$$= \left[-\frac{1}{2}x^2 + \frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\right]_0^1 + \left[\frac{1}{2}x^2 - 2x + \frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\right]_1^2 = \left|-\frac{1}{2} - \frac{2}{\pi}\right| + \left|-2 - \frac{2}{\pi} + \frac{3}{2}\right| = \boxed{1 + \frac{4}{\pi} \approx 2{,}273 \text{ u}^2}$$



9
6280

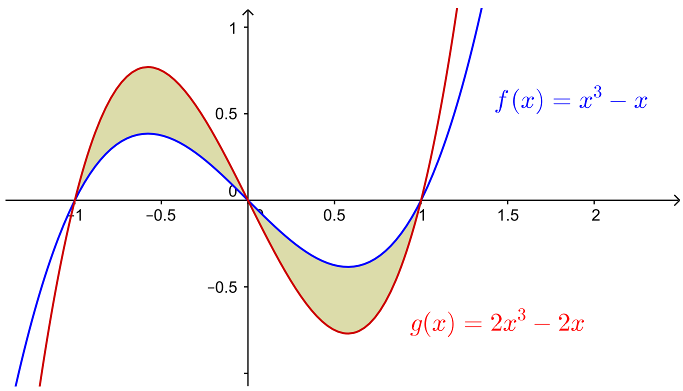
$$\int_0^{\pi^2/4} \frac{\cos\sqrt{x}}{2}\,dx \ ; \ \left\{ \begin{array}{l} t = \sqrt{x} \\ x = t^2 \\ dx = 2t\,dt \end{array} \right\}$$

$$\int \frac{\cos\sqrt{x}}{2}\,dx = \int \frac{\cos t}{2}\,2t\,dt = \int t\cos t\,dt \ ; \ \left\{ \begin{array}{lll} u = t & ; & du = dt \\ dv = \cos t\,dt & ; & v = \int \cos t\,dt = \text{sen}\,t \end{array} \right\}$$

$$\int \frac{\cos\sqrt{x}}{2}\,dx = \int t\cos t\,dt = t\,\text{sen}\,t - \int \text{sen}\,t\,dt = t\,\text{sen}\,t + \cos t = \sqrt{x}\,\text{sen}\sqrt{x} + \cos\sqrt{x}$$

$$\int_0^{\pi^2/4} \frac{\cos\sqrt{x}}{2}\,dx = \left[\sqrt{x}\,\text{sen}\sqrt{x} + \cos\sqrt{x}\right]_0^{\pi^2/4} = \boxed{\frac{\pi}{2} - 1}$$

10
6431



$$\text{área} = \left|\int_{-1}^{0} (g(x) - g(x))\ dx\right| + \left|\int_{0}^{1} (g(x) - g(x))\ dx\right| =$$

$$= \left|\int_{-1}^{0} (x^3 - x)\ dx\right| + \left|\int_{0}^{1} (x^3 - x)\ dx\right| = \left|\left[\tfrac{1}{4}x^4 - \tfrac{1}{2}x^2\right]_{-1}^{0}\right| + \left|\left[\tfrac{1}{4}x^4 - \tfrac{1}{2}x^2\right]_{0}^{1}\right| =$$

$$= \tfrac{1}{4} + \tfrac{1}{4} = \boxed{\tfrac{1}{2}\ \text{u}^2}$$

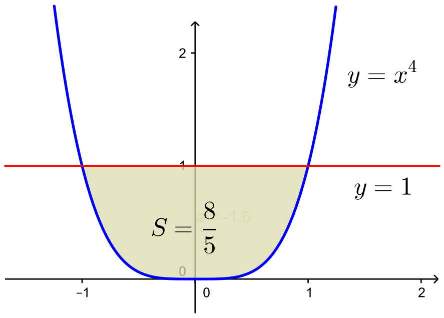
11
6524

Sea $y = k^4$ la recta horizontal buscada. Los puntos de corte con x^4 se obtienen al resolver la ecuación $x^4 = k^4$. Las soluciones son k y $-k$.

$x^4 \geq 0$ por lo que el área coincide con la integral.

$$\int_{-k}^{k} (k^4 - x^4)\ dx = \left[k^4 x - \tfrac{1}{5}x^5\right]_{-k}^{k} = \left(k^5 - \tfrac{1}{5}k^5\right) - \left(-k^5 + \tfrac{1}{5}k^5\right) = \tfrac{8}{5}k^5$$

Por tanto: $\frac{8}{5}k^5 = \frac{8}{5}$ de donde $k = 1$. La recta buscada es $\boxed{y = 1}$



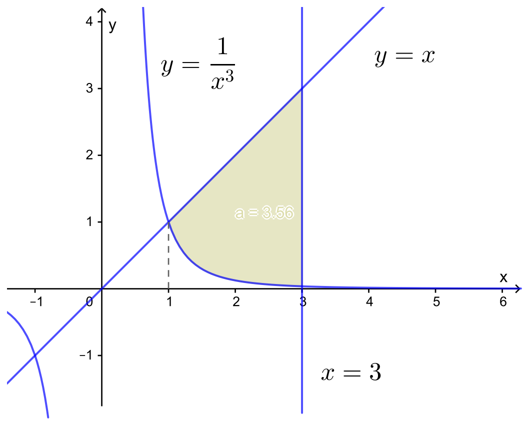
12
6631

$$\int_{1}^{16} \frac{dx}{\sqrt{x} + \sqrt[4]{x}} = \left\{\begin{array}{c} t = \sqrt[4]{x}\ ;\ x = t^4 \\ dx = 4t^3 dt \\ t_1 = \sqrt[4]{x_1} = \sqrt[4]{1} = 1 \\ t_2 = \sqrt[4]{x_2} = \sqrt[4]{16} = 2 \end{array}\right\} = \int_{1}^{2} \frac{4t^3 dt}{t^2 + t} = 4\int_{1}^{2}\left(t - 1 + \frac{1}{t+1}\right) = 4\left[\tfrac{t^2}{2} - t + \ln(t+1)\right]_{1}^{2} =$$

$$= 4\left(\tfrac{2^2}{2} - 2 + \ln(2+1) - \left(\tfrac{1^2}{2} - 1 + \ln(1+1)\right)\right) = 4\left(\ln 3 + \tfrac{1}{2} - \ln 2\right) = \boxed{2 + 4\ln\tfrac{3}{2}}$$

13
6647

a.



b. $S = \displaystyle\int_{1}^{3}\left(x - \frac{1}{x^3}\right) dx = \left[\tfrac{x^2}{2} + \tfrac{1}{2x^2}\right]_{1}^{3} = \tfrac{3^2}{2} + \tfrac{1}{2\cdot 3^2} - \left(\tfrac{1^2}{2} + \tfrac{1}{2\cdot 1^2}\right) = \tfrac{41}{9} - 1 = \boxed{\tfrac{32}{9}\ \text{u}^2}$

El área será menor, pues el punto de corte sigue siendo el $(1{,}1)$ y, para $x > 1$: $\frac{1}{x} > \frac{1}{x^3}$ la "base" del triángulo mixtilíneo se encuentra por encima de la del esbozo del primer apartado, salvo en el punto $(1{,}1)$ donde coinciden.

c. El cálculo es también sencillo:

$$S = \int_{1}^{3}\left(x - \frac{1}{x}\right) dx = \left[\tfrac{x^2}{2} - \ln x\right]_{1}^{3} = \tfrac{3^2}{2} - \ln 3 - \left(\tfrac{1^2}{2} - \ln 1\right) = 4 - \ln 3 \approx 2{,}901\ \text{u}^2$$

14
6752

$f(x) = x^3 + 1$ y $g(x) = x + 1$

Puntos de corte

$$x^3 + 1 = x + 1\ ;\ x^3 - x = 0\ ;\ x\left(x^2 - 1\right) = 0\ ;\ x(x+1)(x-1) = 0;\ x = \left\{\begin{array}{c} -1 \\ 0 \\ 1 \end{array}\right.$$

$$S = \int_{-1}^{0} (f(x) - g(x))\ dx + \int_{0}^{1} (g(x) - f(x))\ dx = \int_{-1}^{0} (x^3 - x)\ dx + \int_{0}^{1} (-x^3 + x)\ dx =$$

$$= \left[\tfrac{x^4}{4} - \tfrac{x^2}{2}\right]_{-1}^{0} + \left[-\tfrac{x^4}{4} + \tfrac{x^2}{2}\right]_{0}^{1} = 0 + \tfrac{1}{4} + \tfrac{1}{4} - 0 = \boxed{\tfrac{1}{2}\ \text{u}^2}$$

15

6889

$y_1 = e^x$; $y_1(0) = 1$; $y_1(1) = e$

a. $y_2 = -x^2$; $y_2(0) = 0$; $y_2(1) = -1$

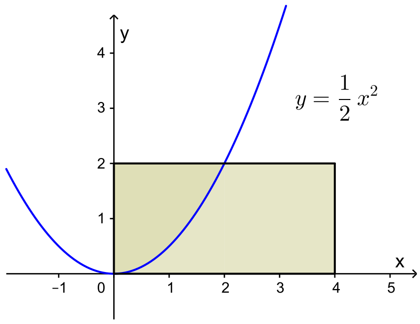
Los puntos pedidos son $\boxed{(0,1)\ ,\ (1,e)\ ,\ (0,0)\ \text{ y }\ (1,-1)}$

Para cualquier x , $e^x > -x^2$, por lo que el área es la integral:

b. $\displaystyle\int_0^1 \left(e^x - (-x^2)\right)\,dx = \int_0^1 \left(e^x + x^2\right)\,dx = \left[e^x + \frac{x^3}{3}\right]_0^1 = e + \frac{1}{3} - e^0 = e + \frac{1}{3} - 1 = \boxed{e - \frac{2}{3}\ u^2}$

16
7083

a.

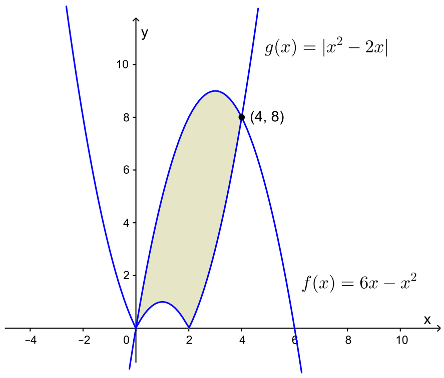


b. $\displaystyle S_1 = \int_0^2 (2 - f(x))\,dx = \int_0^2 \left(2 - \frac{1}{2}x^2\right)\,dx = \left[2x - \frac{x^3}{6}\right]_0^2 = 4 - \frac{8}{6} = \boxed{\frac{8}{3}\ u^2}$

$S_2 = 8 - S_1 = 8 - \frac{8}{3} = \boxed{\frac{16}{3}\ u^2}$

17
7326

a.



$f(x) = 6x - x^2$

$$g(x) = \left|x^2 - 2x\right| = |x(x-2)| = \begin{cases} x^2 - 2x & \text{si } x < 0 \\ -x^2 + 2x & \text{si } 0 \le x \le 2 \\ x^2 - 2x & \text{si } x > 2 \end{cases}$$

b. $\displaystyle S = \int_0^4 (f(x) - g(x))\,dx = \int_0^2 (f(x) - g(x))\,dx + \int_2^4 (f(x) - g(x))\,dx =$

$\displaystyle = \int_0^2 \left(6x - x^2 - \left(-x^2 + 2x\right)\right)\,dx + \int_2^4 \left(6x - x^2 - \left(x^2 - 2x\right)\right)\,dx = \int_0^2 4x\,dx + \int_2^4 \left(-2x^2 + 8x\right)\,dx =$

$\displaystyle = \left[2x^2\right]_0^2 + \left[-\frac{2x^3}{3} + 4x^2\right]_2^4 = 8 - \frac{128}{3} + 64 + \frac{16}{3} - 16 = \boxed{\frac{56}{3}\ u^2}$

18
7590

$f(x)=2x\,\mathrm{e}^{-x}$ y $g(x)=x^2\,\mathrm{e}^{-x}$

Puntos de corte entre las dos curvas

$$x^2\,\mathrm{e}^{-x}=2x\,\mathrm{e}^{-x}\ ;\ x^2\,\mathrm{e}^{-x}-2x\,\mathrm{e}^{-x}=0\ ;\ x(x-2)\,\mathrm{e}^{-x}=0\ ;\ x=\begin{cases}0\\2\end{cases}$$

$$\int_0^2 (g(x)-f(x))\,dx=\int_0^2\left(x^2\,\mathrm{e}^{-x}-2x\,\mathrm{e}^{-x}\right)dx$$

$$\int x\,\mathrm{e}^{-x}\,dx\ :\ \left\{\begin{matrix} u=x & ; & du=dx\\ dv=\mathrm{e}^{-x}\,dx & ; & v=\int \mathrm{e}^{-x}\,dx=-\mathrm{e}^{-x}\end{matrix}\right\}$$

$$\int x\,\mathrm{e}^{-x}\,dx=-x\,\mathrm{e}^{-x}+\int \mathrm{e}^{-x}\,dx=-x\,\mathrm{e}^{-x}-\mathrm{e}^{-x}=-(x+1)\,\mathrm{e}^{-x}$$

$$\int x^2\,\mathrm{e}^{-x}\,dx\ :\ \left\{\begin{matrix} u=x^2 & ; & du=2x\,dx\\ dv=\mathrm{e}^{-x}\,dx & ; & v=\int \mathrm{e}^{-x}\,dx=-\mathrm{e}^{-x}\end{matrix}\right\}$$

$$\int x^2\,\mathrm{e}^{-x}\,dx=-x^2\,\mathrm{e}^{-x}+2\int x\,\mathrm{e}^{-x}\,dx=-x^2\,\mathrm{e}^{-x}+2\left(-x\,\mathrm{e}^{-x}-\mathrm{e}^{-x}\right)=-\left(x^2+2x+2\right)\mathrm{e}^{-x}$$

$$\int\left(x^2\,\mathrm{e}^{-x}-2x\,\mathrm{e}^{-x}\right)dx=\int x^2\,\mathrm{e}^{-x}\,dx-2\int x\,\mathrm{e}^{-x}\,dx=-\left(x^2+2x+2\right)\mathrm{e}^{-x}+2(x+1)\,\mathrm{e}^{-x}=-x^2\mathrm{e}^{-x}$$

$$\int\left(x^2\,\mathrm{e}^{-x}-2x\,\mathrm{e}^{-x}\right)dx=-\left(x^2-1\right)\mathrm{e}^{-x}$$

$$\int_0^2 (g(x)-f(x))\,dx=\int_0^2\left(x^2\,\mathrm{e}^{-x}-2x\,\mathrm{e}^{-x}\right)dx=\left[-x^2\mathrm{e}^{-x}\right]_0^2=-\tfrac{4}{\mathrm{e}^2}\ ;\ \boxed{S=\tfrac{4}{\mathrm{e}^2}\ \mathrm{u}^2}$$

A la vista del resultado obtenido en el cálculo de la primitiva, se descubre que habría sido muy ventajoso el cambio de variable: $x^2\,\mathrm{e}^{-x}=t$, pues así:

$$\int\left(x^2\,\mathrm{e}^{-x}-2x\,\mathrm{e}^{-x}\right)dx\ \left\{\begin{matrix} x^2\,\mathrm{e}^{-x}=t\\ \left(2x\,\mathrm{e}^{-x}-x^2\,\mathrm{e}^{-x}\right)dx=dt\\ \left(x^2\,\mathrm{e}^{-x}-2x\,\mathrm{e}^{-x}\right)dx=-dt\end{matrix}\right\}\ :\ \int\left(x^2\,\mathrm{e}^{-x}-2x\,\mathrm{e}^{-x}\right)dx=\int(-dt)=-t=x^2\,\mathrm{e}^{-x}$$

19
7601

$f(x)=x\cos x\ ;\ x\in\left[0,\frac{\pi}{2}\right]$

$f(x)>0$ en el intervalo $\left[0,\frac{\pi}{2}\right]$ por lo que $S=\int_0^{\pi/2} f(x)\,dx$

$$\int f(x)\,dx=\int x\cos x\,dx\ :\ \left\{\begin{matrix} u=x & ; & du=dx\\ dv=\cos x\,dx & ; & v=\int\cos x\,dx=\operatorname{sen} x\end{matrix}\right\}$$

$$\int x\cos x\,dx=x\operatorname{sen}x-\int\operatorname{sen}x\,dx=x\operatorname{sen}x+\cos x$$

$$S=\left[x\operatorname{sen}x+\cos x\right]_0^{\pi/2}=\tfrac{\pi}{2}-1=\boxed{\tfrac{\pi-2}{2}\ \mathrm{u}^2}$$

20
7699

$f(x)=-x^2+3x$ y $g(x)=\begin{cases}\frac{x}{2} & \text{si}\quad x\le 2\\ 3-x & \text{si}\quad x>2\end{cases}$

$x\le 2\ :\ \frac{x}{2}=-x^2+3x\ ;\ 2x^2-5x=0\ ;\ 2x\left(x-\frac{5}{2}\right)=0\ ;\ x=0$

$f(0)=g(0)=0\ ;\ P(0{,}0)$

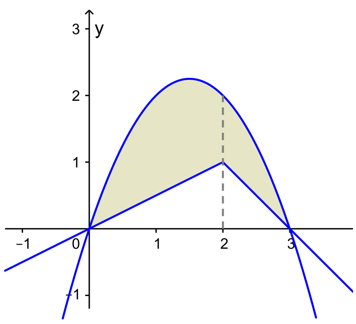
[La otra solución queda fuera de rango]

$x>2\ :\ 3-x=-x^2+3x\ ;\ x^2-4x+3=0\ ;\ (x-1)(x-3)=0\ ;\ x=3$

$f(3)=g(3)=0\ ;\ Q(3{,}0)$

[La otra solución queda fuera de rango]

Los puntos buscados son $\boxed{\begin{cases}P(0{,}0)\\Q(3{,}0)\end{cases}}$

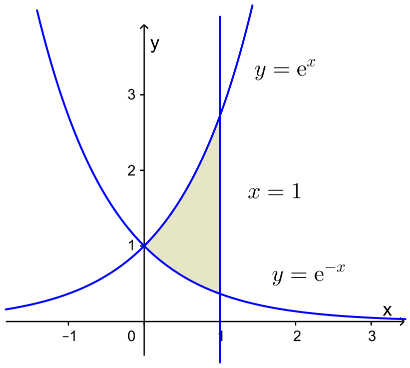


$$S_1=\left|\int_0^2 (f(x)-g(x))\,dx\right|=\left|\int_0^2\left(-x^2+3x-\frac{x}{2}\right)dx\right|=\left|\int_0^2\left(-x^2+\frac{5x}{2}\right)dx\right|=\left|\left[-\tfrac{x^3}{3}+\tfrac{5x^2}{4}\right]_0^2\right|=\tfrac{7}{3}$$

$$S_2=\left|\int_2^3 (f(x)-g(x))\,dx\right|=\left|\int_2^3\left(-x^2+4x-3\right)dx\right|=\left|\left[-\tfrac{x^3}{3}+2x^2-3x\right]_2^3\right|=\tfrac{2}{3}$$

$$S=S_1+S_2=\tfrac{7}{3}+\tfrac{2}{3}=\tfrac{9}{3}=\boxed{3\ \mathrm{u}^2}$$

21
7711

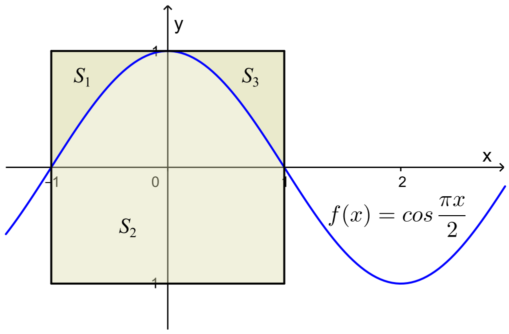


$y=\mathrm{e}^{x},\ y=\mathrm{e}^{-x}$

$$S=\int_0^1\left(\mathrm{e}^{x}-\mathrm{e}^{-x}\right)dx=\left[\mathrm{e}^{x}+\mathrm{e}^{-x}\right]_0^1=\mathrm{e}+\tfrac{1}{\mathrm{e}}-(1+1)=$$

$$=\ \tfrac{\mathrm{e}^2-2\mathrm{e}+1}{\mathrm{e}}\ =\ \boxed{\tfrac{(\mathrm{e}-1)^2}{\mathrm{e}}\approx 1{,}086\ \mathrm{u}^2}$$

22
8117



$f(x) = \cos \frac{\pi x}{2}$

$$S_1 = S_3 = \int_0^1 (1 - f(x))\,dx = \int_0^1 \left(1 - \cos\frac{\pi x}{2}\right) dx = \left[x - \tfrac{2}{\pi}\text{sen}\,\tfrac{\pi x}{2}\right]_0^1 = \boxed{1 - \tfrac{2}{\pi} \approx 0{,}3634\ \text{u}^2}$$

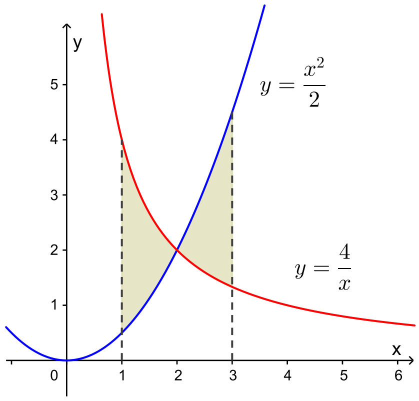
$$S_2 = 2\int_0^1 (f(x) - (-1))\,dx = 2\int_0^1 \left(1 + \cos\frac{\pi x}{2}\right) dx = 2\left[x + \tfrac{2}{\pi}\text{sen}\,\tfrac{\pi x}{2}\right]_0^1 = \boxed{2 + \tfrac{4}{\pi} \approx 3{,}2732\ \text{u}^2}$$

23
8898

a. $y = \frac{x^2}{2}$; $y = \frac{4}{x}$

$\frac{x^2}{2} = \frac{4}{x}$; $x^3 = 8$; $x = 2$, $y = 2$: $\boxed{(2{,}2)}$

b.



c.
$$S = \left|\int_1^2 \left(\frac{4}{x} - \frac{x^2}{2}\right) dx\right| + \left|\int_2^3 \left(\frac{4}{x} - \frac{x^2}{2}\right) dx\right| = \left|\left[4\ln x - \tfrac{x^3}{6}\right]_1^2\right| + \left|\left[4\ln x - \tfrac{x^3}{6}\right]_2^3\right| =$$

$$= 4\ln 2 - \tfrac{7}{6} + \left|4\ln\tfrac{3}{2} - \tfrac{19}{6}\right| = 4\ln 2 - \tfrac{7}{6} - 4\ln\tfrac{3}{2} + \tfrac{19}{6} = \boxed{2 + 4\ln\tfrac{4}{3}\ \text{u}^2 \approx 3{,}1507\ \text{u}^2}$$

24
9255

$f(x) = 5 - x$, $g(x) = \frac{2}{x-2}$

$f(x) = g(x)$; $5 - x = \frac{2}{x-2}$

$$-x^2 + 7x - 10 = 2\ ;\ x^2 - 7x + 12 = 0\ ;\ \begin{cases} x_1 = 3\ ;\ y_1 = 2\ :\ \boxed{P(3{,}2)} \\ x_2 = 4\ ;\ y_2 = 1\ :\ \boxed{Q(4{,}1)} \end{cases}$$

$$S = \left|\int_3^4 (f(x) - g(x))\,dx\right|$$

$$\int_3^4 (f(x) - g(x))\,dx = \int_3^4 \left(-x^2 + 7x - 12\right) dx = \left[-\tfrac{x^3}{3} + 7x^2 - 12x\right]_3^4 = -\tfrac{40}{3} - \left(-\tfrac{27}{2}\right) = \tfrac{1}{6}$$

$$S = \left|\int_3^4 (f(x) - g(x))\,dx\right| = \boxed{\tfrac{1}{6}\ \text{u}^2}$$