22 Escribe en forma de desigualdad y representa los siguientes intervalos:

$P = (1; 2,5)$ $\qquad$ $Q = [-2, 3]$ $\qquad$ $R = [-7, 0]$
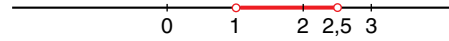
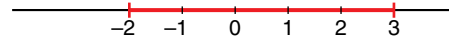$S = [-3, +\infty)$ $\qquad$ $T = (2, +\infty)$ $\qquad$ $I = (-5, 2]$

$P = \{x / 1 < x < 2,5\}$
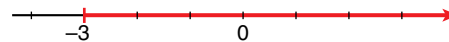
$Q = \{x / -2 \leq x \leq 3\}$
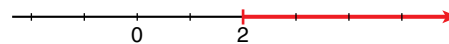
$R = \{x / -7 \leq x \leq 0\}$

$S = \{x / -3 \leq x\}$

$T = \{x / x > 2\}$

$I = \{x / -5 < x \leq 2\}$

Potencias y raíces

23 (ESTÁ RESUELTO EN EL LIBRO).

24 Expresa como potencia única:

a) $\sqrt{3} \sqrt[3]{3}$ $\qquad$ b) $2\sqrt[3]{\dfrac{1}{4}}$ $\qquad$ c) $\dfrac{\sqrt{8}}{\sqrt[3]{4}}$

d) $\dfrac{\sqrt[3]{a^8}}{a^2}$ $\qquad$ e) $\sqrt[3]{\dfrac{1}{a^2}}$ $\qquad$ f) $a\sqrt{\dfrac{1}{a}}$

a) $\sqrt{3} \sqrt[3]{3} = 3^{1/2} \cdot 3^{1/3} = 3^{5/6}$

b) $2\sqrt[3]{\dfrac{1}{4}} = 2\sqrt[3]{\dfrac{1}{2^2}} = 2\dfrac{1}{2^{2/3}} = \dfrac{2}{2^{2/3}} = 2^{1/3}$

c) $\dfrac{\sqrt{8}}{\sqrt[3]{4}} = \dfrac{8^{1/2}}{4^{1/3}} = \dfrac{2^{3/2}}{2^{2/3}} = 2^{5/6}$

d) $\dfrac{\sqrt[3]{a^8}}{a^2} = \dfrac{a^{8/3}}{a^2} = a^{2/3}$

e) $\sqrt[3]{\dfrac{1}{a^2}} = \dfrac{1}{a^{2/3}} = a^{-2/3}$

f) $a\sqrt{\dfrac{1}{a}} = \dfrac{1}{a^{1/2}} = a^{1/2}$

25 Obtén con la calculadora:

a) $\sqrt[5]{9,5^2}$　　　b) $\sqrt[3]{-173}$　　　c) $\sqrt[4]{\left(\frac{14}{9}\right)^3}$　　　d) $\sqrt[4]{5^{-9}}$

e) $28^{3/4}$　　　f) $8^{-1/3}$　　　g) $0,03^{-3/2}$　　　h) $\left(\sqrt[5]{0,0025}\right)^{-1}$

a) $\sqrt[5]{9,5^2} = 9,5^{2/5} \approx 2,46$　　　b) $\sqrt[3]{-173} \approx -5,57$

c) $\sqrt[4]{\left(\frac{14}{9}\right)^3} = \left(\frac{14}{9}\right)^{3/4} \approx 1,39$　　　d) $\sqrt[4]{5^{-9}} = 5^{-9/4} \approx 0,027$

e) $28^{3/4} \approx 12,17$　　　f) $8^{-1/3} = 0,5$

g) $0,03^{-3/2} \approx 192,45$　　　h) $\left(\sqrt[5]{0,0025}\right)^{-1} = (0,0025)^{-1/5} \approx 3,31$

26 Expresa en forma exponencial:

a) $\sqrt[3]{x^2}$　　　b) $\left(\sqrt[5]{a^2}\right)^3$　　　c) $\sqrt[8]{a^5 \cdot a^2}$　　　d) $\sqrt[3]{\sqrt[4]{x}}$

e) $\left(\sqrt{a}\right)^{-3}$　　　f) $\sqrt[6]{a^3}$　　　g) $\left(\sqrt[4]{a^2}\right)^2$　　　h) $\sqrt[5]{a^{10}}$

a) $x^{2/3}$　　　b) $\left(a^{2/5}\right)^3 = a^{6/5}$

c) $\sqrt[8]{a^7} = a^{7/8}$　　　d) $\sqrt[12]{x} = x^{1/12}$

e) $\left(a^{1/2}\right)^{-3} = a^{-3/2}$　　　f) $a^{3/6} = a^{1/2}$

g) $\left(a^{2/4}\right)^2 = a$　　　h) $a^{10/5} = a^2$

27 Expresa como una raíz:

a) $15^{1/2}$　　　b) $(a^2)^{1/3}$　　　c) $(x^{-1})^{5/4}$　　　d) $(a^{1/5})^{-4}$

e) $(a^{2/3})^{1/2}$　　　f) $a^2 \cdot a^{1/2}$　　　g) $(3^{-2/5})^{10/3}$

a) $\sqrt{15}$　　　b) $\sqrt[3]{a^2}$

c) $\sqrt[4]{x^{-5}}$　　　d) $\sqrt[5]{a^{-4}}$

e) $a^{1/3} = \sqrt[3]{a}$　　　f) $a^{2 + 1/2} = a^{5/2} = \sqrt{a^5}$

g) $3^{(-2/5) \cdot (10/3)} = 3^{-4/3} = \sqrt[3]{3^{-4}}$

28 Expresa como potencia única:

a) $\dfrac{\sqrt[3]{a^7}}{a^4}$　　　b) $\sqrt[4]{\dfrac{1}{a}}$　　　c) $\dfrac{\sqrt{125}}{\sqrt[3]{25}}$

d) $\dfrac{1}{2}\sqrt{2}\sqrt[4]{2}$　　　e) $\dfrac{\sqrt[3]{a^2}}{a\sqrt{a}}$　　　f) $\dfrac{\sqrt[3]{a^2}}{a^2} \cdot \dfrac{a^3}{\sqrt{a}}$

a) $\dfrac{\sqrt[3]{a^7}}{a^4} = \dfrac{a^{7/3}}{a^4} = a^{-5/3}$

b) $\sqrt[4]{\dfrac{1}{a}} = \sqrt[4]{a^{-1}} = a^{-1/4}$

c) $\dfrac{\sqrt{125}}{\sqrt[3]{25}} = \dfrac{\sqrt{5^3}}{\sqrt[3]{5^2}} = \dfrac{5^{3/2}}{5^{2/3}} = 5^{5/6}$

d) $\dfrac{1}{2} \sqrt{2} \sqrt[4]{2} = \dfrac{1}{2} \sqrt[4]{2^2 \cdot 2} = \dfrac{1}{2} 2^{3/4} = \dfrac{2^{3/4}}{2} = 2^{-1/4}$

e) $\dfrac{\sqrt[3]{a^2}}{a \sqrt{a}} = \dfrac{a^{2/3}}{a \cdot a^{1/2}} = \dfrac{a^{2/3}}{a^{3/2}} = a^{-5/6}$

f) $\dfrac{\sqrt[3]{a^2}}{a^2} \cdot \dfrac{a^3}{\sqrt{a}} = \dfrac{a^{2/3} \cdot a^3}{a^2 \cdot a^{1/2}} = \dfrac{a^{11/3}}{a^{5/2}} = a^{7/6}$

Radicales

29 Multiplica y simplifica el resultado:

a) $\sqrt{2a} \ \sqrt{3a} \ \sqrt{6a}$

b) $\sqrt[3]{a} \ \sqrt[3]{a^2} \ \sqrt[3]{b^4} \ \sqrt[3]{b^2}$

c) $\sqrt{5a} \ \sqrt{10ab} \ \sqrt{8a^3b} \ \sqrt{a}$

a) $\sqrt{2a} \ \sqrt{3a} \ \sqrt{6a} = \sqrt{36a^3} = 6a\sqrt{a}$

b) $\sqrt[3]{a} \ \sqrt[3]{a^2} \ \sqrt[3]{b^4} \ \sqrt[3]{b^2} = \sqrt[3]{a^3b^6} = ab^2$

c) $\sqrt{5a} \ \sqrt{10ab} \ \sqrt{8a^3b} \ \sqrt{a} = \sqrt{5 \cdot 10 \cdot 8 \cdot a^6b^2} =$

$$= \sqrt{5^2 \cdot 2^4 \cdot a^6b^2} = 5 \cdot 4 \cdot a^3b = 20a^3b$$

30 Simplifica los siguientes radicales:

a) $\sqrt[6]{5^3}$ b) $\sqrt[15]{2^{12}}$ c) $\sqrt[10]{a^8}$

d) $\sqrt[12]{a^4 \cdot b^8}$ e) $\sqrt[8]{(x^2y^2)^2}$ d) $\sqrt[3]{\sqrt[4]{x^5 \cdot x^7}}$

a) $\sqrt[6]{5^3} = \sqrt{5}$ b) $\sqrt[15]{2^{12}} = \sqrt[5]{2^4}$

c) $\sqrt[10]{a^8} = \sqrt[5]{a^4}$ d) $\sqrt[12]{a^4b^8} = \sqrt[3]{a \cdot b^2}$

e) $\sqrt[8]{(x^2 \cdot y^2)^2} = \sqrt[4]{x^2 \cdot y^2} = \sqrt{x \cdot y}$ f) $\sqrt[3]{\sqrt[4]{x^5x^7}} = \sqrt[12]{x^5x^7} = \sqrt[12]{x^{12}} = x$

31 Extrae factores de los siguientes radicales:

a) $\sqrt[3]{16x^6}$

b) $\sqrt{\dfrac{28x^5}{75y^3}}$

c) $(\sqrt{\sqrt{2}})^{10}$

d) $\sqrt{\dfrac{8a^5}{b^4}}$

e) $\sqrt[4]{\dfrac{25a^2b}{c^6}}$

f) $\sqrt{\dfrac{32a^3}{45b^4}}$

a) $\sqrt[3]{16x^6} = \sqrt[3]{2^4x^6} = 2\sqrt[3]{2} \cdot x^2$

b) $\sqrt{\dfrac{28x^5}{75y^3}} = \sqrt{\dfrac{2^2 \cdot 7x^5}{5^2 \cdot 3y^3}} = \dfrac{2x^2}{5y} \cdot \sqrt{\dfrac{7x}{3y}}$

c) $(\sqrt{\sqrt{2}})^{10} = \sqrt[4]{2^{10}} = 2^2\sqrt[4]{2^2} = 4\sqrt{2}$

d) $\sqrt{\dfrac{8a^5}{b^4}} = \sqrt{\dfrac{2^3a^5}{b^4}} = \dfrac{2a^2}{b^2} \cdot \sqrt{2a}$

e) $\sqrt[4]{\dfrac{25a^2b}{c^6}} = \sqrt[4]{\dfrac{5^2a^2b}{c^6}} = \dfrac{1}{c} \cdot \sqrt[4]{\dfrac{5^2a^2b}{c^2}} = \dfrac{1}{c} \cdot \sqrt[4]{\dfrac{25a^2b}{c^2}}$

f) $\sqrt{\dfrac{32a^3}{45b^4}} = \sqrt{\dfrac{2^5 \cdot a^3}{3^2 \cdot 5 \cdot b^4}} = \dfrac{2^2a}{3b^2}\sqrt{\dfrac{2a}{5}} = \dfrac{4a}{3b^2}\sqrt{\dfrac{2a}{5}}$

32 Reduce a índice común y ordena de menor a mayor:

a) $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{4}$, $\sqrt[5]{5}$, $\sqrt[6]{6}$

b) $\sqrt[3]{2^4}$, $\sqrt[4]{5^3}$, $\sqrt[6]{3^5}$

a) $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{4}$, $\sqrt[5]{5}$, $\sqrt[6]{6}$ $\rightarrow$ $\sqrt[60]{2^{30}}$, $\sqrt[60]{3^{20}}$, $\sqrt[60]{4^{15}}$, $\sqrt[60]{5^{12}}$, $\sqrt[60]{6^{10}}$

Se observa que $2^{30} = 4^{15}$ $\rightarrow$ $\sqrt{2} = \sqrt[4]{4}$

Comparamos los radicandos $\rightarrow$ $6^{10} < 5^{12} < 2^{30} < 3^{20}$

Luego, $\sqrt[6]{6} < \sqrt[5]{5} < \sqrt{2} = \sqrt[4]{4} < \sqrt[3]{3}$.

b) $\sqrt[3]{2^4}$, $\sqrt[4]{5^3}$, $\sqrt[6]{3^5}$ $\rightarrow$ $\sqrt[12]{2^{16}}$, $\sqrt[12]{5^9}$, $\sqrt[12]{3^{10}}$

Comparamos los radicandos $\rightarrow$ $3^{10} < 2^{16} < 5^9$

Luego, $\sqrt[6]{3^5} < \sqrt[3]{2^4} < \sqrt[4]{5^3}$.

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33 Introduce dentro de la raíz y simplifica:

a) $2\sqrt{\dfrac{3}{2}}$

b) $3\sqrt{\dfrac{2}{3}}$

c) $2\sqrt[3]{\dfrac{1}{4}}$

d) $2\sqrt[4]{\dfrac{5}{12}}$

e) $\dfrac{1}{2}\sqrt{12}$

f) $\dfrac{2}{3}\sqrt[3]{\dfrac{9}{4}}$

a) $2\sqrt{\dfrac{3}{2}} = \sqrt{\dfrac{2^2 \cdot 3}{2}} = \sqrt{6}$

b) $3\sqrt{\dfrac{2}{3}} = \sqrt{\dfrac{3^2 \cdot 2}{3}} = \sqrt{6}$

c) $2\sqrt[3]{\dfrac{1}{4}} = \sqrt[3]{\dfrac{2^3}{4}} = \sqrt[3]{2}$

d) $2\sqrt[4]{\dfrac{5}{12}} = \sqrt[4]{\dfrac{2^4 \cdot 5}{12}} = \sqrt[4]{\dfrac{2^2 \cdot 5}{3}} = \sqrt[4]{\dfrac{20}{3}}$

e) $\dfrac{1}{2}\sqrt{12} = \sqrt{\dfrac{12}{2^2}} = \sqrt{3}$

f) $\dfrac{2}{3}\sqrt[3]{\dfrac{9}{4}} = \sqrt[3]{\dfrac{2^3}{3^3} \cdot \dfrac{9}{4}} = \sqrt[3]{\dfrac{2}{3}}$

34 **Divide y simplifica el resultado:**

a) $\dfrac{\sqrt{12}}{\sqrt{3}}$ b) $\dfrac{\sqrt[3]{4}}{\sqrt{2}}$ c) $\sqrt[4]{\dfrac{5}{12}} : \sqrt[4]{\dfrac{20}{3}}$

d) $\dfrac{\sqrt[4]{a}}{\sqrt[4]{ab}}$ e) $\sqrt{\dfrac{3}{2}} : \sqrt{\dfrac{2}{3}}$ f) $\dfrac{\sqrt[6]{20}}{\sqrt[4]{10}}$

a) $\dfrac{\sqrt{12}}{\sqrt{3}} = \sqrt{\dfrac{12}{3}} = \sqrt{4} = 2$

b) $\dfrac{\sqrt[3]{4}}{\sqrt{2}} = \dfrac{\sqrt[6]{4^2}}{\sqrt[6]{2^3}} = \sqrt[6]{\dfrac{4^2}{2^3}} = \sqrt[6]{2}$

c) $\sqrt[4]{\dfrac{5}{12}} : \sqrt[4]{\dfrac{20}{3}} = \sqrt[4]{\dfrac{5}{12} : \dfrac{20}{3}} = \sqrt[4]{\dfrac{5 \cdot 3}{12 \cdot 20}} = \sqrt[4]{\dfrac{1}{16}} = \dfrac{1}{2}$

d) $\dfrac{\sqrt[4]{a}}{\sqrt[4]{ab}} = \sqrt[4]{\dfrac{a}{ab}} = \sqrt[4]{\dfrac{1}{b}}$

e) $\sqrt{\dfrac{3}{2}} : \sqrt{\dfrac{2}{3}} = \sqrt{\dfrac{3}{2} : \dfrac{2}{3}} = \sqrt{\dfrac{3^2}{2^2}} = \dfrac{3}{2}$

f) $\dfrac{\sqrt[6]{20}}{\sqrt[4]{10}} = \dfrac{\sqrt[12]{20^2}}{\sqrt[12]{10^3}} = \sqrt[12]{\dfrac{400}{1\,000}} = \sqrt[12]{\dfrac{4}{10}}$

35 (ESTÁ RESUELTO EN EL LIBRO).

Unidad 1. El número real

36 Suma:

a) $\sqrt{3} + \dfrac{3\sqrt{3}}{4} - \dfrac{5\sqrt{3}}{3}$ 　　　　　　b) $2\sqrt{8} + 4\sqrt{72} - 7\sqrt{18}$

c) $3\sqrt{2} + 4\sqrt{8} - \sqrt{32} + \sqrt{50}$ 　　　d) $5\sqrt{12} + \sqrt{27} - 8\sqrt{75} + \sqrt{48}$

a) $\sqrt{3} + \dfrac{3\sqrt{3}}{4} - \dfrac{5\sqrt{3}}{3} = \left(1 + \dfrac{3}{4} - \dfrac{5}{3}\right)\sqrt{3} = \dfrac{\sqrt{3}}{12}$

b) $2\sqrt{8} + 4\sqrt{72} - 7\sqrt{18} = 4\sqrt{2} + 4 \cdot 3 \cdot 2\sqrt{2} - 7 \cdot 3\sqrt{2} =$
$$= (4 + 24 - 21)\sqrt{2} = 7\sqrt{2}$$

c) $3\sqrt{2} + 4\sqrt{8} - \sqrt{32} + \sqrt{50} = 3\sqrt{2} + 8\sqrt{2} - 4\sqrt{2} + 5\sqrt{2} =$
$$= (3 + 8 - 4 + 5)\sqrt{2} = 12\sqrt{2}$$

d) $5\sqrt{12} + \sqrt{27} - 8\sqrt{75} + \sqrt{48} = 10\sqrt{3} + 3\sqrt{3} - 40\sqrt{3} + 4\sqrt{3} =$
$$= (10 + 3 - 40 + 4)\sqrt{3} = -23\sqrt{3}$$

37 Efectúa:

a) $\sqrt{320} + \sqrt{80} - \sqrt{500}$ 　　　　b) $\sqrt[3]{54} - \sqrt[3]{2}$

c) $\sqrt{\dfrac{7}{64}} + \sqrt{\dfrac{7}{4}}$ 　　　　　　　d) $\sqrt[5]{96} - \sqrt[5]{\dfrac{3}{32}}$

e) $\sqrt{150} + \sqrt{54} - \sqrt{24}$ 　　　　f) $\sqrt[3]{\dfrac{135}{8}} - \sqrt[3]{\dfrac{5}{8}}$

a) $\sqrt{320} + \sqrt{80} - \sqrt{500} = \sqrt{2^6 \cdot 5} + \sqrt{2^4 \cdot 5} - \sqrt{2^2 \cdot 5^3} =$
$$= 2^3 \cdot \sqrt{5} + 2^2 \cdot \sqrt{5} - 10\sqrt{5} =$$
$$= 8\sqrt{5} + 4\sqrt{5} - 10\sqrt{5} = 2\sqrt{5}$$

b) $\sqrt[3]{54} - \sqrt[3]{2} = \sqrt[3]{3^3 \cdot 2} - \sqrt[3]{2} = 3\sqrt[3]{2} - \sqrt[3]{2} = 2 \cdot \sqrt[3]{2}$

c) $\sqrt{\dfrac{7}{64}} + \sqrt{\dfrac{7}{4}} = \dfrac{1}{8}\sqrt{7} + \dfrac{1}{2}\sqrt{7} = \dfrac{5}{8}\sqrt{7}$

d) $\sqrt[5]{96} - \sqrt[5]{\dfrac{3}{32}} = \sqrt[5]{2^5 \cdot 3} - \dfrac{1}{2}\sqrt[5]{3} = 2\sqrt[5]{3} - \dfrac{1}{2}\sqrt[5]{3} = \dfrac{3}{2}\sqrt[5]{3}$

e) $\sqrt{150} + \sqrt{54} - \sqrt{24} = \sqrt{5^2 \cdot 2 \cdot 3} + \sqrt{2 \cdot 3^3} - \sqrt{2^3 \cdot 3} =$
$$= 5\sqrt{6} + 3\sqrt{6} - 2\sqrt{6} = 6\sqrt{6}$$

f) $\sqrt[3]{\dfrac{135}{8}} - \sqrt[3]{\dfrac{5}{8}} = \sqrt[3]{\dfrac{3^3 \cdot 5}{2^3}} - \sqrt[3]{\dfrac{5}{2^3}} = \dfrac{3}{2}\sqrt{5} - \dfrac{1}{2}\sqrt{5} = \sqrt{5}$

38 **Racionaliza y simplifica:**

a) $\dfrac{2}{\sqrt{2}}$ b) $\dfrac{4}{\sqrt{6}}$ c) $\dfrac{6}{\sqrt{12}}$ d) $\dfrac{3}{\sqrt{15}}$

a) $\dfrac{2}{\sqrt{2}} = \dfrac{2\sqrt{2}}{2} = \sqrt{2}$ b) $\dfrac{4}{\sqrt{6}} = \dfrac{4\sqrt{6}}{6} = \dfrac{2\sqrt{6}}{3}$

c) $\dfrac{6}{\sqrt{12}} = \dfrac{6\sqrt{12}}{12} = \dfrac{\sqrt{12}}{2} = \dfrac{2\sqrt{3}}{2} = \sqrt{3}$ d) $\dfrac{3}{\sqrt{15}} = \dfrac{3\sqrt{15}}{15} = \dfrac{\sqrt{15}}{5}$

39 **Racionaliza:**   a) $\dfrac{1 + \sqrt{6}}{2\sqrt{3}}$ b) $\dfrac{3\sqrt{6} + 2\sqrt{2}}{3\sqrt{3} + 2}$

a) Multiplicamos el numerador y denominador por $\sqrt{3}$.

$$\dfrac{1 + \sqrt{6}}{2\sqrt{3}} = \dfrac{(1 + \sqrt{6})\,\sqrt{3}}{2\sqrt{3}\,\sqrt{3}} = \dfrac{\sqrt{3} + \sqrt{18}}{2 \cdot 3} = \dfrac{\sqrt{3} + 3\sqrt{2}}{6}$$

b) Multiplicamos numerador y denominador por $3\sqrt{3} - 2$.

$$\dfrac{3\sqrt{6} + 2\sqrt{2}}{3\sqrt{3} + 2} = \dfrac{(3\sqrt{6} + 2\sqrt{2})\,(3\sqrt{3} - 2)}{(3\sqrt{3} + 2)\,(3\sqrt{3} - 2)} = \dfrac{9\sqrt{18} - 6\sqrt{6} + 6\sqrt{6} - 4\sqrt{2}}{(3\sqrt{3})^2 - 2^2} =$$

$$= \dfrac{9\sqrt{18} - 4\sqrt{2}}{27 - 4} = \dfrac{27\sqrt{2} - 4\sqrt{2}}{23} = \dfrac{23\sqrt{2}}{23} = \sqrt{2}$$

40 **Racionaliza:**

a) $\dfrac{3}{\sqrt[3]{5}}$ b) $\dfrac{1}{\sqrt[8]{a^5}}$ c) $\dfrac{8}{\sqrt{5} - 1}$ d) $\dfrac{\sqrt{3}}{\sqrt{2} + \sqrt{3}}$

a) $\dfrac{3}{\sqrt[3]{5}} = \dfrac{3\,\sqrt[3]{5^2}}{\sqrt[3]{5}\,\sqrt[3]{5^2}} = \dfrac{3\,\sqrt[3]{5^2}}{\sqrt[3]{5^3}} = \dfrac{3\sqrt[3]{5^2}}{5}$

b) $\dfrac{1}{\sqrt[8]{a^5}} = \dfrac{\sqrt[8]{a^3}}{\sqrt[8]{a^5}\,\sqrt[8]{a^3}} = \dfrac{\sqrt[8]{a^3}}{\sqrt[8]{a^8}} = \dfrac{\sqrt[8]{a^3}}{a}$

c) $\dfrac{8}{\sqrt{5} - 1} = \dfrac{8(\sqrt{5} + 1)}{(\sqrt{5} - 1)\,(\sqrt{5} + 1)} = \dfrac{8(\sqrt{5} + 1)}{\sqrt{5}^2 - 1^2} = \dfrac{8(\sqrt{5} + 1)}{5 - 1} =$

$= \dfrac{8(\sqrt{5} + 1)}{4} = 2(\sqrt{5} + 1)$

d) $\dfrac{\sqrt{3}}{\sqrt{2} + \sqrt{3}} = \dfrac{\sqrt{3}(\sqrt{2} - \sqrt{3})}{(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})} = \dfrac{\sqrt{6} - 3}{2 - 3} = 3 - \sqrt{6}$

41 **Racionaliza y simplifica:**

a) $\dfrac{2}{1 + \sqrt{2}}$ b) $\dfrac{14}{3 - \sqrt{2}}$ c) $\dfrac{23}{5 - \sqrt{2}}$

d) $\dfrac{1 + \sqrt{3}}{1 - \sqrt{3}}$ e) $\dfrac{11}{2\sqrt{5} + 3}$ f) $\dfrac{\sqrt{3} + 2\sqrt{2}}{\sqrt{3} - 2\sqrt{2}}$

g) $\dfrac{10}{2\sqrt{3} - \sqrt{2}}$ h) $\dfrac{\sqrt{2}}{2\sqrt{2} + 3}$ i) $\dfrac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$

a) $\dfrac{2}{1 + \sqrt{2}} = \dfrac{2(1 - \sqrt{2})}{(1 + \sqrt{2})(1 - \sqrt{2})} = \dfrac{2(1 - \sqrt{2})}{1 - 2} = \dfrac{2(1 - \sqrt{2})}{-1} = 2(\sqrt{2} - 1)$

b) $\dfrac{4}{3 - \sqrt{2}} = \dfrac{14(3 + \sqrt{2})}{(3 - \sqrt{2})(3 + \sqrt{2})} = \dfrac{4(3 + \sqrt{2})}{9 - 2} = 2(3 + \sqrt{2})$

c) $\dfrac{23}{5 - \sqrt{2}} = \dfrac{23(5 + \sqrt{2})}{(5 - \sqrt{2})(5 + \sqrt{2})} = \dfrac{23(5 + \sqrt{2})}{25 - 2} = 5 + \sqrt{2}$

d) $\dfrac{1 + \sqrt{3}}{1 - \sqrt{3}} = \dfrac{(1 + \sqrt{3})^2}{(1 - \sqrt{3})(1 + \sqrt{3})} = \dfrac{1 + 3 + 2\sqrt{3}}{1 - 3} = \dfrac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$

e) $\dfrac{11}{2\sqrt{5} + 3} = \dfrac{11(2\sqrt{5} - 3)}{(2\sqrt{5} + 3)(2\sqrt{5} - 3)} = \dfrac{11(2\sqrt{5} - 3)}{20 - 9} = \dfrac{11(2\sqrt{5} - 3)}{11} = 2\sqrt{5} - 3$

f) $\dfrac{\sqrt{3} + 2\sqrt{2}}{\sqrt{3} - 2\sqrt{2}} = \dfrac{(\sqrt{3} + 2\sqrt{2})(\sqrt{3} + 2\sqrt{2})}{(\sqrt{3} - 2\sqrt{2})(\sqrt{3} + 2\sqrt{2})} = \dfrac{3 + 8 + 4\sqrt{6}}{3 - 8} =$

$= \dfrac{11 + 4\sqrt{6}}{-5} = -\dfrac{11 + 4\sqrt{6}}{5}$

g) $\dfrac{10}{2\sqrt{3} - \sqrt{2}} = \dfrac{10(2\sqrt{3} + \sqrt{2})}{(2\sqrt{3} - \sqrt{2})(2\sqrt{3} + \sqrt{2})} = \dfrac{10(2\sqrt{3} + \sqrt{2})}{12 - 2} =$

$= \dfrac{10(2\sqrt{3} + \sqrt{2})}{10} = 2\sqrt{3} + \sqrt{2}$

h) $\dfrac{\sqrt{2}}{2\sqrt{2} + 3} = \dfrac{\sqrt{2}(2\sqrt{2} - 3)}{(2\sqrt{2} + 3)(2\sqrt{2} + 3)} = \dfrac{4 - 3\sqrt{2}}{8 - 9} = 3\sqrt{2} - 4$

i) $\dfrac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \dfrac{(\sqrt{5} - \sqrt{3})(\sqrt{5} - \sqrt{3})}{(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})} = \dfrac{(\sqrt{5})^2 + (\sqrt{3})^2 - 2\sqrt{5}\sqrt{3}}{(\sqrt{5})^2 - (\sqrt{3})^2} =$

$= \dfrac{5 + 3 - 2\sqrt{15}}{5 - 3} = \dfrac{8 - 2\sqrt{15}}{2} = 4 - \sqrt{15}$