

BT2-1 Ejercicio 1

4ESO

$$a. P(x) + Q(x) = (3x^4 - 6x^3 + 4x - 2) + (x^3 - 2x^2 - 3x + 1) = 3x^4 - 6x^3 + 4x - 2 + x^3 - 2x^2 - 3x + 1 = 3x^4 - 5x^3 - 2x^2 + x - 1$$

$$b. 2P(x) - 3Q(x) + 4R(x) = 2 \cdot (3x^4 - 6x^3 + 4x - 2) - 3(x^3 - 2x^2 - 3x + 1) + 4(2x^2 + 4x - 5) = 6x^4 - 12x^3 + 8x - 4 - 3x^3 + 6x^2 + 9x - 3 + 8x^2 + 16x - 20 = 6x^4 - 15x^3 + 14x^2 + 33x - 27$$

$$c. 3[P(x) \cdot Q(x) - 2S(x)] = 3[(3x^4 - 6x^3 + 4x - 2) \cdot (x^3 - 2x^2 - 3x + 1) - 2(x^2 + 1)] = 3[3x^7 - 6x^6 - 9x^5 + 3x^4 - 6x^6 + 12x^5 + 12x^4 - 6x^3 + 4x^4 - 2x^3 - 12x^2 + 4x - 2x^3 + 4x^2 + 6x - 2 - 2x^2 - 2] = 3[3x^7 - 12x^6 + 3x^5 + 25x^4 - 16x^3 - 10x^2 + 6x - 4] = 9x^7 - 36x^6 + 9x^5 + 75x^4 - 48x^3 - 30x^2 + 18x - 12$$

$$d. P(x) \cdot S(x) - R(x) = (3x^4 - 6x^3 + 4x - 2) \cdot (x^2 + 1) - (2x^2 + 4x - 5) = 3x^6 + 3x^4 - 6x^5 - 6x^3 + 4x^3 + 4x - 2x^2 - 2 - 2x^2 - 4x + 5 = 3x^6 - 6x^5 + 3x^4 - 2x^3 - 4x^2 + 3$$

Ejercicio 2

$$a. (3x^5 + 2x^4 - 7x^3 + 2x - 3) : (x^2)$$

$3x^5 + 2x^4 - 7x^3$	$+ 2x - 3$	x^2
$- 3x^5$		$3x^3 + 2x^2 - 7x$
$2x^4 - 7x^3$	$+ 2x - 3$	
$- 2x^4$		
$- 7x^3$	$+ 2x - 3$	
$7x^3$		
	$2x - 3$	

$$(3x^5 + 2x^4 - 7x^3 + 2x - 3) = x^2 \cdot (3x^3 + 2x^2 - 7x) + (2x - 3)$$

$$b. (3x^4 + 5x^3 - 2x + 3) : (x^2 - 3x + 2)$$

$3x^4 + 5x^3$	$- 2x + 3$	$x^2 - 3x + 2$
$- 3x^4 + 9x^3 - 6x^2$		$3x^2 + 14x + 36$
$14x^3 - 6x^2 - 2x + 3$		
$- 14x^3 + 42x^2 - 28x$		
	$36x^2 - 30x + 3$	
	$- 36x^2 + 108x - 72$	
	$78x - 69$	

$$(3x^4 + 5x^3 - 2x + 3) = (x^2 - 3x + 2) \cdot (3x^2 + 14x + 36) + (78x + 69)$$

$$c. (3x^5 - 2x^3 + 7x^2 - 2x) : (x^3 + 3x^2 - 1) =$$

$$\begin{array}{r} 3x^5 \quad -2x^3 + 7x^2 - 2x \quad |x^3 + 3x^2 - 1 \\ \underline{-3x^5 - 9x^4} \quad \quad \quad + 3x^2 \quad \quad \quad 3x^2 - 9x + 19 \end{array}$$

$$-9x^4 - 2x^3 + 10x^2 - 2x$$

$$\underline{9x^4 + 27x^3} \quad \quad \quad -9x$$

$$19x^3 + 10x^2 - 11x$$

$$\underline{-19x^3 - 57x^2} \quad \quad \quad + 19$$

$$-4x^2 - 11x + 19$$

$$(3x^5 - 2x^3 + 7x^2 - 2x) = (x^3 + 3x^2 - 1)(3x^2 - 9x + 19) + (-4x^2 - 11x + 19)$$

$$d. (4x^4 - 5x^3 + x - 2) : (x^2 - 2x + 1) =$$

$$\begin{array}{r} 4x^4 - 5x^3 \quad \quad \quad + x - 2 \quad |x^2 - 2x + 1 \\ \underline{-4x^2 + 8x^3 - 4x^2} \quad \quad \quad 4x^2 + 3x + 2 \end{array}$$

$$3x^3 - 4x^2 + x - 2$$

$$\underline{-3x^3 + 6x^2 - 3x}$$

$$2x^2 - 2x - 2$$

$$\underline{-2x^2 + 4x - 2}$$

$$2x - 4$$

$$(4x^4 - 5x^3 + x - 2) = (x^2 - 2x + 1)(4x^2 + 3x + 2) + (2x - 4)$$

$$e. (x^5 - 5x^4 + 20x^2 - 16x) : (x^2 - 2x + 8) =$$

$$\begin{array}{r} x^5 - 5x^4 \quad \quad \quad + 20x^2 - 16x \quad |x^2 - 2x + 8 \\ \underline{-x^5 + 2x^4 - 8x^3} \quad \quad \quad x^3 - 3x^2 - 14x + 16 \end{array}$$

$$-3x^4 - 8x^3 + 20x^2 - 16x$$

$$\underline{3x^4 - 6x^3 + 24x^2}$$

$$-14x^3 + 44x^2 - 16x$$

$$\underline{14x^3 - 28x^2 + 112x}$$

$$16x^2 + 96x$$

$$\underline{-16x^2 + 32x - 128}$$

$$128x - 128$$

$$(x^5 - 5x^4 + 20x^2 - 16x) = (x^2 - 2x + 8)(x^3 - 3x^2 - 14x + 16) + (128x - 128)$$

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$$f. (x^4 - 1) : (x^2 - 1) = x^2 + 1$$

$$(x^4 - 1) = (x^2 - 1)(x^2 + 1)$$

Ejercicio 3

a. $(5x^3 - 3x^2 + x - 2) : (x - 2)$

$$\begin{array}{r|rrrr} 5 & -2 & 1 & -2 \\ 2 & 10 & 16 & 34 \\ \hline & 5 & 8 & 17 & \underline{32} \end{array} \quad \left| \quad 5x^3 - 3x^2 + x - 2 = (x - 2) \cdot (5x^2 + 8x + 17) + 32 \right.$$

b. $(x^4 - 5x^3 + 7x + 3) : (x + 1)$

$$\begin{array}{r|rrrrr} 1 & -5 & 0 & 7 & 3 \\ -1 & -1 & 6 & -6 & -1 \\ \hline & 1 & -6 & 6 & 1 & \underline{2} \end{array} \quad \left| \quad x^4 - 5x^3 + 7x + 3 = (x + 1)(x^3 - 6x^2 + 6x + 1) + 2 \right.$$

c. $(-x^3 + 4x) : (x - 3)$

$$\begin{array}{r|rrrr} -1 & 0 & 4 & 0 \\ 3 & -3 & -9 & -15 \\ \hline & -1 & -3 & -5 & \underline{-15} \end{array} \quad \left| \quad (-x^3 + 4x) = (x - 3) \cdot (-x^2 - 3x - 5) + (-15) \right.$$

d. $(x^4 - 3x^3 + 5) : (x + 2)$

$$\begin{array}{r|rrrrr} 1 & -3 & 0 & 0 & 5 \\ -2 & -2 & 10 & -20 & 40 \\ \hline & 1 & -5 & 10 & -20 & \underline{45} \end{array} \quad \left| \quad (x^4 - 3x^3 + 5) = (x + 2)(x^3 - 5x^2 + 10x - 20) + 45 \right.$$

Ejercicio 4

m/ $P(x) = x^3 - mx^2 + 5x - 2$ divisible por $(x + 1)$

$P(x)$ divisible por $(x + 1) \Rightarrow$ división exacta \Rightarrow resto 0 \Rightarrow
 $\Rightarrow x = -1$ raíz $\Rightarrow P(-1) = 0$

$$P(-1) = (-1)^3 - m(-1)^2 + 5(-1) - 2 = -1 - m - 5 - 2 = 0 \Rightarrow$$
$$\Rightarrow -m - 8 = 0 \Rightarrow \underline{\underline{m = -8}}$$

Ejercicio 5

a/ $x = 3$ raíz de $Q(x) = x^3 - 6x^2 + ax - 2$

$x = 3$ raíz de $Q(x) \Rightarrow Q(3) = 0$

$$Q(3) = 3^3 - 6 \cdot 3^2 + a \cdot 3 - 2 = 27 - 54 + 3a - 2 = 0 \Rightarrow$$

$$\Rightarrow 3a - 29 = 0 \Rightarrow a = \frac{29}{3}$$

Ejercicio 6

$$a. 4x^2 + 8x + 4 = (2x + 2)^2$$

$$b. x^2 - 6x + 9 = (x - 3)^2$$

$$c. 9x^2 - 36 = (3x - 6)(3x + 6)$$

$$d. a^2 - 2a + 1 = (a - 1)^2$$

$$e. x^2 + 2xy + y^2 = (x + y)^2$$

$$f. a^2 - 16 = (a - 4)(a + 4)$$

Ejercicio 7

$$a. 3x^3y + 6xy^2 - 9x^2y^3 = 3xy(x^2 + 2y - 3xy^2)$$

$$b. 2 + 4a(b+2) + b = 4a(b+2) + b + 2 = (b+2) \cdot 4a + (b+2) = (b+2)(4a+1)$$

$$c. (x^4 + 2x^3 + 4x^2 + 8x)^2 = ((x^4 + 4x^2) + (2x^3 + 8x))^2 = (x^2(x^2 + 4) + 2x(x^2 + 4))^2 = (x^2 + 4)(x^2 + 2x))^2 = (x(x^2 + 4)(x + 2))^2 = x^2 \cdot (x^2 + 4)^2 \cdot (x + 2)^2$$

$$d. 2a^2b + 6ab + 3a + 9 = 2ab(a + 3) + 3(a + 3) = (a + 3)(2ab + 3)$$

$$e. 21a^5b^4 - 18a^4b^6 = 3a^4b^4(7a - 6b^2)$$

$$f. 10a - 5a^2 + 2b - ab = 10a + 2b - 5a^2 - ab = 2(5a + b) - a(5a + b) = (5a + b)(2 - a)$$

$$g. x^4 + x^3 - x - 1 = x^3(x + 1) - (x + 1) = (x + 1)(x^3 - 1)$$

$$h. a^3 - 3a^2 - 9a + 27 = a^2(a - 3) - 9(a - 3) = (a - 3)(a^2 - 9) =$$

Ejercicio 8.

$$a. 9x^2 - 25 = (3x - 5)(3x + 5) = 3(x - \frac{5}{3}) \cdot 3(x + \frac{5}{3}) = 9(x - \frac{5}{3})(x + \frac{5}{3})$$

Roots: $x = \frac{5}{3}$

$x = -\frac{5}{3}$

b. $4x^6 + 12x^3 + 9 = (2x^3 + 3)^2 = \left[\left(x + \sqrt[3]{\frac{3}{2}}\right) \cdot \left(2x^2 - 2\sqrt[3]{\frac{3}{2}}x + 2\sqrt[3]{\frac{3}{2}}\right) \right]^2$

$2x^3 + 3 = 0 \Rightarrow x^3 = -\frac{3}{2}$

$x = \sqrt[3]{-\frac{3}{2}} = -\sqrt[3]{\frac{3}{2}}$ raíz única de

Raíz: $x = -\sqrt[3]{\frac{3}{2}}$ (doble)

	2	0	0	3
$\sqrt[3]{\frac{3}{2}}$		$-2\sqrt[3]{\frac{3}{2}}$	$2\sqrt[3]{\left(\frac{3}{2}\right)^2}$	$2\sqrt[3]{\left(\frac{3}{2}\right)^3} = -2 \cdot \frac{3}{2} = -3$
	2	$-2\sqrt[3]{\frac{3}{2}}$	$2\sqrt[3]{\left(\frac{3}{2}\right)^2}$	0

c. $x^3 + x^2 - x - 1 = (x-1)(x^2 + 2x + 1) = (x-1)(x+1)^2$

$x=1$ es raíz:

Raíces: $\begin{cases} x=1 \\ x=-1 \text{ (doble)} \end{cases}$

	1	1	-1	-1
1		1	2	1
	1	2	1	0

d. $x^3 - 2x^2 + x = x(x^2 - 2x + 1) = x \cdot (x-1)^2$

Raíces: $\begin{cases} x=0 \\ x=1 \text{ (doble)} \end{cases}$

e. $x^3 - 2x^2 + 2x - 4 = (x-2) \cdot (x^2 + 2)$

$x=2$ es raíz:

Raíz: $x=2$

	1	-2	2	-4
2		2	0	4
	1	0	2	0

f. $2x^3 - 2x^2 + x - 1 = (x-1) \cdot (2x^2 + 1)$

$x=1$ es raíz:

Raíz: $x=1$

	2	-2	1	-1
1		2	0	1
	2	0	1	0

Ejercicio 9

a. $\frac{x^3}{x^2 + x^3} = \frac{x^3}{x^2(1+x)} = \frac{x}{x+1}$

b. $\frac{x-1}{x^2-1} = \frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)} = \frac{1}{x+1}$

c. $\frac{x+1}{x^2+2x+1} = \frac{\cancel{x+1}}{(x+1)^2} = \frac{1}{x+1}$

Ejercicio 10

$$a. \frac{x^2+2}{x^3-1} - \frac{2x^2+1}{(x-1)^2} = \frac{x^2+2}{(x-1)(x^2+x+1)} - \frac{2x^2+1}{(x-1)^2} =$$

$$x^3-1 = (x-1) \cdot (x^2+x+1) \quad \left| \begin{array}{l} = \frac{(x^2+2)(x-1)}{(x-1)^2(x^2+x+1)} - \frac{(2x^2+1)(x^2+x+1)}{(x-1)^2(x^2+x+1)} = \\ x=1 \text{ es raíz:} \end{array} \right.$$

$$\begin{array}{r|rrrr} 1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 0 \end{array}$$

$$x^2+x+1=0$$

$$x = \frac{-1 \pm \sqrt{1-4}}{2} \rightarrow \text{sol}$$

$$\begin{aligned} & \frac{x^3+2x-x^2-2-(2x^4+2x^3+2x^2+x^2+x+1)}{(x-1)^2(x^2+x+1)} \\ & = \frac{-2x^4-x^3-4x^2+x-3}{x^4-x^3-x+1} \end{aligned}$$

$$b. \frac{x^2-4}{x^3+1} : \frac{x^3-8}{x+1} = \frac{(x+2)(x-2)}{(x+1)(x^2-x+1)} : \frac{(x-2)(x^2+2x+4)}{x+1} =$$

$$x^3+1 = (x+1) \cdot (x^2-x+1)$$

$$\begin{array}{r|rrrr} 1 & 0 & 0 & 1 \\ -1 & -1 & 1 & -1 \\ \hline 1 & -1 & 1 & 0 \end{array}$$

$$x^3-8 = (x-2) \cdot (x^2+2x+4)$$

$$\begin{array}{r|rrrr} 1 & 0 & 0 & -8 \\ 2 & 2 & 4 & 8 \\ \hline 1 & 2 & 4 & 0 \end{array}$$

$$\begin{aligned} & = \frac{(x+2)\cancel{(x-2)}(x+1)}{\cancel{(x+1)}(x^2-x+1)\cancel{(x-2)}(x^2+2x+4)} = \\ & = \frac{x+2}{(x^2-x+1)(x^2+2x+4)} = \\ & = \frac{x+2}{x^4+x^3+3x^2-2x+4} \end{aligned}$$

$$c. \frac{3x^2-5x+2}{x^3+1} \cdot \frac{x^2+5x+4}{2x^2-3x+1} = \frac{3\cancel{(x-1)}(x-\frac{2}{3})}{\cancel{(x+1)}(x^2-x+1)} \cdot \frac{(x+4)\cancel{(x+1)}}{2\cancel{(x-1)}(x-\frac{1}{2})} =$$

$$3x^2-5x+2=0$$

$$x = \frac{5 \pm \sqrt{25-24}}{2 \cdot 3} =$$

$$= \frac{5 \pm 1}{6} = \begin{cases} \frac{4}{6} = \frac{2}{3} \\ \frac{6}{6} = 1 \end{cases}$$

$$x^2+5x+4=0$$

$$x = \frac{-5 \pm \sqrt{25-16}}{2} =$$

$$= \frac{-5 \pm 3}{2} = \begin{cases} \frac{-2}{2} = -1 \\ \frac{-8}{2} = -4 \end{cases}$$

$$2x^2-3x+1=0$$

$$x = \frac{3 \pm \sqrt{9-8}}{2 \cdot 2} =$$

$$= \frac{3 \pm 1}{4} = \begin{cases} \frac{4}{4} = 1 \\ \frac{2}{4} = \frac{1}{2} \end{cases}$$

$$\begin{aligned} & = \frac{(3x-2)(x+4)}{(x^2-x+1)(2x-1)} = \\ & = \frac{3x^2+10x-8}{2x^3-3x^2+3x-1} \end{aligned}$$

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$$d. \frac{x^2+x+1}{x^3-x^2+x-1} = \frac{x+1}{x-1} = \frac{(x^2+x+1) \cdot \cancel{(x-1)}}{\cancel{(x-1)}(x^2+1)(x+1)} = \frac{x^2+x+1}{(x^2+1)(x+1)}$$

$$x^3-x^2+x-1 = (x-1)(x^2+1)$$

$$\left. \begin{array}{r|rrrr} 1 & -1 & 1 & -1 \\ 1 & 1 & 0 & 1 \\ \hline 1 & 0 & 1 & 0 \end{array} \right\} = \frac{x^2+x+1}{x^3+2x^2+x+1}$$

Ejercicio 11

$$\left(\frac{a^2+b^2}{a^2-b^2} - \frac{a+b}{a-b} \right) \cdot \frac{a+b}{ab} = \left(\frac{a^2+b^2}{(a+b)(a-b)} - \frac{(a+b)^2}{(a+b)(a-b)} \right) \cdot \frac{a+b}{ab}$$

$$= \frac{a^2+b^2 - (a^2+b^2+2ab)}{(a+b)(a-b)} \cdot \frac{a+b}{ab} = \frac{-2ab}{(a-b)ab} = -\frac{2}{a-b}$$

$$\frac{x^2-x+9}{x^3-9x} + \frac{1}{x^2-9} - \frac{1}{x-3} + \frac{1}{x} =$$

$$= \frac{x^2-x+9}{x \cdot (x-3)(x+3)} + \frac{1}{(x-3)(x+3)} - \frac{1}{x-3} + \frac{1}{x} =$$

$$= \frac{x^2-x+9}{x(x-3)(x+3)} + \frac{x}{x(x-3)(x+3)} - \frac{x \cdot (x+3)}{x(x-3)(x+3)} + \frac{(x-3) \cdot (x+3)}{x(x-3)(x+3)} =$$

$$= \frac{x^2-x+9+x-x^2-3x+x^2-9}{x(x-3)(x+3)} = \frac{x^2-3x}{x(x-3)(x+3)}$$

$$= \frac{\cancel{x}(x-3)}{x \cdot \cancel{(x-3)}(x+3)} = \frac{1}{x+3}$$

$$\left(1 - \frac{1}{x}\right) : \frac{3x-3}{x^6} + \frac{1}{x} = \frac{x-1}{x} : \frac{3x-3}{x^6} + \frac{1}{x} =$$

$$= \frac{x^5 \cdot \cancel{(x-1)}}{x \cdot 3 \cdot \cancel{(x-1)}} + \frac{1}{x} = \frac{x^5}{3} + \frac{1}{x} = \frac{x^6+3}{3x}$$