1 a) $AX + 3B = B(A^t + 3I)$

$AX = B(A^t + 3I) - 3B$

$AX = BA^t + 3BI - 3B$

$AX = BA^t + 3B - 3B$

$\boxed{AX = BA^t}$

Si $|A| \neq 0 \Rightarrow \exists A^{-1} \Rightarrow \boxed{X = A^{-1} B A^t}$

b) para $A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \Rightarrow$

$|A| = 4 - (0 + 3 + 0) = 4 - 3 = 1 \neq 0 \Rightarrow \exists A^{-1}$

Cálculo de A^{-1}

$$\text{Adj} = \begin{pmatrix} \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} -1 & 3 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} \\ -\begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \\ \begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ -1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 4-3 & -(-2) & -1 \\ 0 & 2 & -1 \\ 0 & -3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & -1 \\ 0 & 2 & -1 \\ 0 & -3 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 2 & -1 \\ 0 & -3 & 2 \end{pmatrix}^T \underset{\substack{\uparrow \\ |A| = 1}}{=} \begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & -3 \\ -1 & -1 & 2 \end{pmatrix} \checkmark$$

Cálculo de X

$$X = A^{-1} B A^t = A^{-1} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 0 & 3 & 2 \end{pmatrix} = A^{-1} \begin{pmatrix} 0 & 3 & 2 \\ 0 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & -3 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 0 & 3 & 2 \\ 0 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 0 & 3 & 2 \\ -3 & 13 & 6 \\ 2 & -7 & -3 \end{pmatrix}} \leftarrow \text{solución}$$

$\boxed{2}$

$$\left.\begin{array}{l} x+y = 1 \\ (a-1)y + z = 0 \\ x + ay + (a-1)z = a \end{array}\right\} \quad \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & a-1 & 1 & 0 \\ 1 & a & a-1 & a \end{array}\right)$$

P.2

en A, tomo el menor de orden 2 $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \neq 0 \Rightarrow r(A) \geq 2$

calculo $|A| = (a-1)^2 + 1 - (a) = a^2 - 2a + 1 + 1 - a = \boxed{a^2 - 3a + 2}$ ✓

$a^2 - 3a + 2 = 0 \quad ; \quad a = \dfrac{3 \pm \sqrt{9 - 4 \cdot 2}}{2} = \dfrac{3 \pm 1}{2} \begin{cases} \boxed{2} \\ \boxed{1} \end{cases}$

* Si $a \neq 2$ y $a \neq 1 \Rightarrow r(A) = 3 = r(A^*) = n^o$ incógnitas $\Rightarrow$ S.C.D. ($\exists$! sol.)
  $r(A^*) \geq r(A)$

* Si $a = 2$ ó $a = 1$ (en ese caso $r(A) = 2$)

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & a-1 & 1 & 0 \\ 1 & a & a-1 & a \end{array}\right)$$

int. $C_2 \leftrightarrow C_4$

tengo que calcular el rango (A^*)

$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & a & a-1 \end{vmatrix} = 1 - a$$

* Si $a = 2 \Rightarrow$ $\det = 1 - 2 = -1 \neq 0 \Rightarrow r(A^*) = 3 \neq 2 = r(A)$
  $\Rightarrow$ S.I. ($\nexists$ solución)

* Si $a = 1 \Rightarrow$ $\det = 1 - 1 = 0 \Rightarrow r(A^*) = 2 = r(A) < n^o$ incóg
  $\Rightarrow$ S.C. Ind. (∞ soluc)

b) $a = 1$ S.C.Indet (∞ soluc)

$$\left(\begin{array}{ccc|c} \cancel{1} & \cancel{1} & \cancel{0} & \cancel{1} \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right)$$

Esta fila está repetida, no añade información al sistema.

$$\left.\begin{array}{r} z = 0 \\ x + y = 1 \end{array}\right\} \quad \begin{array}{l} z = 0 \\ x = \lambda \\ y = 1 - \lambda \end{array}$$

Soluciones $\{(\lambda, 1-\lambda, 0) \mid \lambda \in \mathbb{R}\}$

3 a) $f(x) = \begin{cases} ax^2+b & x<3 \\ \ln(x-2) & x\geq 3 \end{cases}$ deriv en $\mathbb{R}$

Continua y derivable en $\mathbb{R} - \{3\}$: f función a trozos

$f_1(x) = ax^2+b$ func. polinómica $\Rightarrow$ cont y derivable en todo $\mathbb{R}$ en particular, lo es en $(-\infty, 3)$

$f_2(x) = \ln(x-2)$ func. logarítmica $\Rightarrow$ cont y deriv en su DOMINIO $(2, +\infty)$ en particular lo es en $(3, +\infty) \subset (2, +\infty)$

Sólo falta ver lo que ocurre en $x=3$

f continua en 3

$$\left.\begin{array}{l} \lim_{x\to 3^-} f(x) = \lim_{x\to 3} ax^2+b = 9a+b \\ \lim_{x\to 3^+} f(x) = \lim_{x\to 3} \ln(x-2) = \ln(3-2) = 0 \end{array}\right\} \Rightarrow \boxed{9a+b=0}$$

$\lim_{x\to 3^+} f(x) = f(3)$

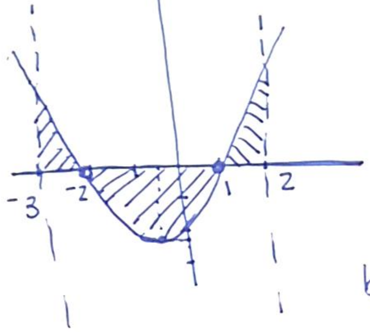
f derivable en 3

$$\left.\begin{array}{l} f_1'(x) = 2ax \rightarrow \lim_{x\to 3^-} 2ax = 6a \\ f_2'(x) = \frac{1}{(x-2)} \rightarrow \lim_{x\to 3} \frac{1}{x-2} = \frac{1}{3-2} = 1 \end{array}\right\} \Rightarrow \boxed{6a=1}$$

$\Rightarrow \boxed{a = \frac{1}{6}}$ $\quad b = -9a = -\frac{9}{6} = \boxed{-\frac{3}{2}}$

b) $f(x) = (x-1)(x+2)$ parábola $\cup$ convexa con ptos de corte $x=1$, $x=-2$, vértice en $x = -\frac{1}{2}$, $y = -2'25$

$f(x) = x^2+2x-x-2 = x^2+x-2$



(P.9)

$A = A_1 + A_2 + A_3$

$$A = \underbrace{\int_{-3}^{-2} f(x)\,dx}_{A_1} + \underbrace{\left(-\int_{-2}^{1} f(x)\,dx\right)}_{A_2} + \underbrace{\int_{1}^{2} f(x)\,dx}_{A_3}$$

busco una primitiva para $F(x) = \int f(x)dx =$

$$= \int (x-1)(x+2)\,dx = \int (x^2+x-2)\,dx = \frac{x^3}{3} + \frac{x^2}{2} - 2x + k$$

$$A = F(-2) - F(-3) - \left[F(1) - F(-2)\right] + F(2) - F(1) =$$

$$= F(-2) - F(-3) - F(1) + F(-2) + F(2) - F(1) =$$

$$= 2F(-2) - F(-3) - 2F(1) + F(2) =$$

$$= 2\left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(-\frac{27}{3} + \frac{9}{2} + 6\right) - 2\left(\frac{1}{3} + \frac{1}{2} - 2\right) + \left(\frac{8}{3} + \frac{4}{2} - 4\right)$$

$$= -\frac{16}{3} + 12 + 9 - \frac{9}{2} - 6 - \frac{2}{3} - 1 + 4 + \frac{8}{3} - 2 =$$

$$= 16 - \frac{10}{3} - \frac{9}{2} = \frac{96-20-27}{6} = \boxed{\frac{49}{6}}\,u^2$$

$\boxed{4}$ a) $f(x) = \dfrac{4x^2+3x+4}{x}$ Dom $f = \mathbb{R} - \{0\}$

i) Asíntotas A.V. en $x=0$

$$\begin{cases} \lim\limits_{x\to 0^-} \dfrac{4x^2+3x+4}{x} = -\infty \\ \lim\limits_{x\to 0^+} \dfrac{4x^2+3x+4}{x} = +\infty \end{cases}$$

Como $\partial P = 2 = \partial Q + 1$ tenemos A.O.

$$m = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{4x^2+3x+4}{1\cdot x^2} \overset{(*)}{=} \frac{4}{1} = \boxed{4}$$

(*) (por ser F.A. $= \dfrac{P(x)}{Q(x)}$ con $\partial P = \partial Q$)

$$n = \lim_{x\to\infty} f(x) - mx = \lim_{x\to\infty} \frac{4x^2+3x+4}{x} - 4x = \lim_{x\to\infty} \frac{3x+4}{x} = \boxed{3}$$

(P5)

[4] a) continuación $\Rightarrow$ $y = 4x + 3$ A. Oblicua de $f(x)$ cuando $x \to +\infty$

por ser $f(x)$ F. Algebraica $\Rightarrow$ $y = 4x+3$ también es A. Oblicua cuando $x \to -\infty$

ii) Monotonía $f'(x) = \frac{(8x+3)x - (4x^2+3x+4)}{x^2} =$

$= \frac{8x^2 + \cancel{3x} - 4x^2 - \cancel{3x} - 4}{x^2} = \frac{4x^2-4}{x^2} = \frac{4(x^2-1)}{x^2}$

$f'(x) = 0 \Rightarrow 4(x^2-1) = 0 \Leftrightarrow x^2 - 1 = 0 \Leftrightarrow x = \pm 1$

como $Dom f = \mathbb{R} - \{0\}$

y $x \in Dom f$

$x \neq 0$

Estudiamos el signo de $f'(x)$

$\oplus \nearrow$ -1 $\ominus \searrow$ 0 $\ominus \searrow$ 1 $\oplus \nearrow$

en -1^-, -2, $f'(-2) = +$

en -1^+, signo $f'(-0'9) = -$

en 1^-, signo $f'(0'9) = -$

en 1^+, signo $f'(1'1) = +$

f creciente en $(-\infty, -1) \cup (1, +\infty)$

f decreciente en $(-1, 0) \cup (0, 1)$ $(0 \notin Dom f)$

en $(-1, -5)$ alcanza máximo relativo

y en el pto $(1, 11)$ alcanza mínimo relativo

b) $\lim_{x\to 0} \frac{sen^2 x - 3x^2}{e^{x^2} - \cos 2x} = \frac{0-0}{1-1} = \frac{0}{0}$ IND

$f(x) = sen^2 x - 3x^2$ func. cont y deriv en 0

$g(x) = e^{x^2} - \cos 2x$ cont y deriv en 0

podemos aplicar L'Hopital

$\lim_{x\to 0} \frac{2 \, senx \cos x - 6x}{e^{x^2} \cdot 2x + 2 sen 2x} = \lim_{x \to 0} \frac{sen 2x - 6x}{2(xe^{x^2} + sen 2x)} = \frac{0}{0}$ IND

Volvemos a estar en condiciones de aplicar L'Hôpital (P. 6.)

$$\lim_{x\to 0} \frac{(\operatorname{sen} 2x - 6x)'}{\left(2\left(xe^{x^2}+\operatorname{sen} 2x\right)\right)'} = \lim_{x\to 0} \frac{2\cos 2x - 6}{2\left(e^{x^2}+xe^{x^2}\cdot 2x + 2\cos 2x\right)} =$$

$$= \frac{2-6}{2(1+2)} = \frac{-4}{6} = \boxed{-\frac{2}{3}}$$ $\Rightarrow$ por L'Hôpital

$$\exists \lim_{x\to 0} \frac{\operatorname{sen}^2 x - 3x^2}{e^{x^2}-\cos 2x} = \boxed{-\frac{2}{3}}$$

— o —

$\boxed{5}$ a) $\pi_1: x+y-z+2=0$

$\vec{n}_{\pi_1}(1,1,-1)$

$$\pi_2: \begin{cases} x = 2+\lambda+\mu \\ y = \lambda+3\mu \\ z = -1-\lambda \end{cases}$$

$$\left.\begin{matrix} P_2(2,0,-1) \\ \vec{v}_1(1,1,-1) \\ \vec{v}_2(1,3,0) \end{matrix}\right\} \text{Ec. general de } \pi_2$$

$$\begin{vmatrix} x-2 & y & z+1 \\ 1 & 1 & -1 \\ 1 & 3 & 0 \end{vmatrix} = 0$$

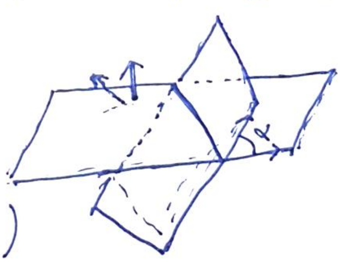
$\pi_2: \boxed{3x-y+2z-4=0}$

$\vec{n}_{\pi_2}(3,-1,2)$

posic. relativa

$$\underbrace{\frac{1}{3} \neq \frac{1}{-1} \neq \frac{-1}{2}}_{\Downarrow} = \frac{2}{-4}$$ (comparamos $\frac{A}{A'}, \frac{B}{B'}, \frac{C}{C'}, \frac{D}{D'}$)

π_1 y π_2 son secantes



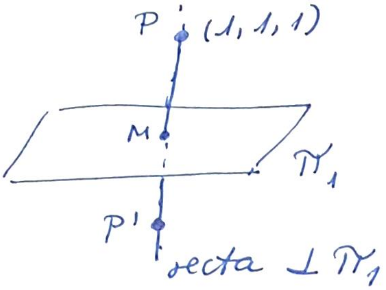
b i) ángulo (π_1, π_2) = ángulo $(\vec{n}_{\pi_1}, \vec{n}_{\pi_2})$

$$= \arccos \frac{|\vec{n}_{\pi_1}\cdot\vec{n}_{\pi_2}|}{|\vec{n}_{\pi_1}||\vec{n}_{\pi_2}|} = \arccos \frac{|(1,1,-1)\cdot(3,-1,2)|}{\sqrt{3}\cdot\sqrt{9+1+4}} =$$

$$= \arccos \frac{|3-1-2|}{\sqrt{3}\,\sqrt{14}} = \arccos 0 = 90^\circ$$ (son perpendiculares)

b ii) r, $P_r(1,1,1)$ $\vec{d}_r \perp \pi_1$; $\vec{d}_r = \vec{n}_{\pi_1}(1,1,-1)$

Simétrico de $(1,1,1)$ respecto de π_1



r recta $\begin{cases} P_r(1,1,1) \\ \vec{d}_r(1,1,-1) \end{cases}$

$r \equiv \begin{cases} x = 1 + \lambda \\ y = 1 + \lambda \\ z = 1 - \lambda \end{cases}$

$M = r \cap \pi_1$ $\qquad \pi_1 : x + y - z + 2 = 0$

$(1+\lambda) + (1+\lambda) - (1-\lambda) + 2 = 0$

$\cancel{1} + \lambda + \cancel{1} + \lambda - \cancel{1} + \lambda + 2 = 0$

$3 + 3\lambda = 0$

$3\lambda = -3$

$\lambda = -1$

$\Rightarrow M \begin{cases} x = 1 + (-1) = 0 \\ y = 1 + (-1) = 0 \\ z = 1 - (-1) = 2 \end{cases}$ $\qquad M(0,0,2)$

M pto medio P y P' $\Rightarrow$ $M = \dfrac{P+P'}{2} \Rightarrow 2M = P + P'$

$\Rightarrow P' = 2M - P = 2(0,0,2) - (1,1,1) = (0,0,4) - (1,1,1)$

$= \boxed{(-1,-1,3)}$ simétrico de P con resp. π_1

$\boxed{6}$ a) λ? $\quad \left.\begin{matrix} A(3,0,-1) \\ B(2,2,-1) \\ D(\lambda,6,-1) \end{matrix}\right\}$ Sean coplanarios (3 ptos siempre son coplanarios)

$\left.\begin{matrix} \vec{AB} \\ \vec{AD} \end{matrix}\right\}$ vectores directores del plano
A pto. del plano.

$\vec{AB} = B - A = (-1, 2, 0)$

$\vec{AD} = D - A = (\lambda - 3, 6, 0)$

$\vec{n}_\pi = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 0 \\ \lambda-3 & 6 & 0 \end{vmatrix} =$

$= \vec{i}(0) - \vec{j}(0) + \vec{k}(-\cancel{6} - 2\lambda + \cancel{6})$

$= (0, 0, -2\lambda)$

$\pi : 0x + 0y - 2\lambda z + D = 0$

$-2\lambda z + D = 0$

como $A(3,0,-1) \in \pi$ $\qquad +2\lambda + D = 0$

$\Rightarrow D = -2\lambda$

$\boxed{\pi : -2\lambda z - 2\lambda = 0}$

si $\lambda \neq 0$ $\boxed{\pi: \ -2z-2=0}$

A, B y $D \in \pi \quad \forall \lambda \neq 0$

¿qué ocurre si $\lambda = 0$?

$\left.\begin{array}{l} A(3,0,-1) \\ B(2,2,-1) \\ C(0,6,-1) \end{array}\right\}$ siguen $\in \pi$

$-2z-2=0$

sólo que en ese caso $\left.\begin{array}{l} \vec{AB}(-1,2,0) \\ \vec{AD}(-3,6,0) \end{array}\right\}$ serían proporcionales

y los 3 ptos estarían alineados (no habría 1 único plano sino ∞)

6 b) $\left.\begin{array}{l} P(-4,4,2) \\ Q(4,8,-4) \end{array}\right\} r = \begin{cases} x = -4+8\lambda \\ y = 4+4\lambda \\ z = 2-6\lambda \end{cases}$ piden $r \cap \pi$

$\vec{PQ} = Q-P =$
$= (4,8,-4) - (-4,4,2)$
$= (4+4, 8-4, -4-2)$
$= (8,4,-6)$

$\pi: 4x+2y-3z-15=0$

$4(-4+8\lambda) + 2(4+4\lambda) - 3(2-6\lambda) - 15 = 0$

$-16 + 32\lambda + 8 + 8\lambda - 6 + 18\lambda - 15 = 0$

$-29 + 58\lambda = 0$

$\lambda = \frac{29}{58} = \boxed{\frac{1}{2}}$

$\Rightarrow r \cap \pi =$ pto $\left(-4+8\frac{1}{2}, 4+4\frac{1}{2}, 2-6\frac{1}{2}\right) = \boxed{(0,6,-1)}$

$\boxed{7}$

F (0'3): $H \to 0'6 = P(H/F)$; $M \to 0'4 = P(M/F)$

$\bar{F}$ (0'7): $H \to 0'3 = P(H/\bar{F})$; $M \to 0'7 = P(M/\bar{F})$

a) $P(M) = P(M \cap F) + P(M \cap \bar{F}) = P(M/F)\,P(F) + P(M/\bar{F}) \cdot P(\bar{F}) =$

$= 0'4 \cdot 0'3 + 0'7 \cdot 0'7 = \boxed{0'61} = \boxed{61\%}$

b) $P(F/H) \underset{\text{Bayes}}{=} \frac{P(F \cap H)}{P(H)} = \frac{P(H \cap F)}{P(H)} \underset{\text{Bayes}}{=} \frac{P(H/F)\,P(F)}{P(H)} =$

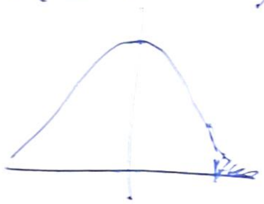
$= \frac{0'6 \cdot 0'3}{1-0'61} \simeq \underline{\underline{0'4615}}$

$\boxed{3}$ $\quad Z \in N(1220, 120) \qquad z' = \dfrac{Z-1220}{120} \in N(0,1)$ P.9

a) $P(Z > 1400) = P\left(Z' > \dfrac{1400-1220}{120}\right) =$

$= P(Z' > 1'5) = 1 - P(Z' < 1'5) = 1 - 0'9332 = \boxed{0'0668}$

$\boxed{6'68\%}$



b) 980 €/dia $\qquad P(Z < 980) = P\left(Z' < \dfrac{980-1220}{120}\right)$

$= P(Z' < -2) = P(Z' > 2) = 1 - P(Z' < 2) = 1 - 0'9772$

$= 0'0228 = \boxed{2'28\%}$