1) a) $f(x)=\dfrac{2x^3-x^2-x}{x^2-1}$

ESTUDAMOS O DOMINIO:

$x^2-1=0 \Leftrightarrow x=\pm 1 \Rightarrow Dom(f)=\mathbb{R}-\{\pm 1\} \Rightarrow$ A FUNCIÓN É CONTINUA EN $\mathbb{R}-\{\pm 1\}$ 0'2

CONTINUIDADE EN $x=1$:

$$\lim_{x\to 1}\frac{2x^3-x^2-x}{x^2-1}=\left[\frac{0}{0}\right]=\lim_{x\to 1}\frac{x(2x^2-x-1)}{(x+1)(x-1)}=\lim_{x\to 1}\frac{x\cdot 2\cdot\cancel{(x-1)}(x+\frac{1}{2})}{(x+1)\cancel{(x-1)}}$$

$$2x^2-x-1=0 \Leftrightarrow x=\frac{1\pm\sqrt{1+8}}{4}=\frac{1\pm 3}{4}\begin{cases}x=1\\ x=-\frac{1}{2}\end{cases}$$

$$=\lim_{x\to 1}\frac{2x(x+\frac{1}{2})}{x+1}=\frac{2\cdot(1+\frac{1}{2})}{1+1}=\frac{3}{2} \text{ 0'1} \Rightarrow \exists \lim_{x\to 1} f(x)$$

0'2

PERO $\nexists f(1)$ 0'1 $\Rightarrow$ A FUNCIÓN É DESCONTINUA EVITABLE EN $x=1$.

CONTINUIDADE EN $x=-1$:

$$\lim_{x\to -1}\frac{2x^3-x^2-x}{x^2-1}=\frac{-2-1+1}{0}=\frac{-2}{0}=-\infty \text{ 0'2}$$

$$\lim_{x\to -1^-} f(x)=\frac{+}{-}=-\infty$$

$$\lim_{x\to -1^+} f(x)=\frac{+}{+}=-\infty$$

$\Rightarrow$ EN $x=-1$ É DESCONTINUA INEVITABLE DE SALTO INFINITO. 0'2

b) $f(x)=16-x^2$ ① vértice $\left.\begin{array}{l} x_v=0 \\ y_v=16 \end{array}\right\} \Rightarrow (0,16)$ 0'1

② $x=0 \Rightarrow y=16$

$y=0 \Rightarrow 0=16-x^2 \Rightarrow x=\pm 4 \Rightarrow (4,0), (-4,0)$

③ $a=-1<0 \Rightarrow \cap$ cóncava

$g(x)=(x+2)^2-4=x^2+4x+4-4=x^2+4x$ ① VÉRTICE $\begin{array}{l} x_v=\frac{-4}{2}=-2 \\ y_v=4-8=-4 \end{array} \Rightarrow$

$(-2,-4)$ 0'1

② $x=0 \Rightarrow y=0 \Rightarrow (0,0)$

$y=0 \Rightarrow 0=x^2+4x$

$0=x(x+4) \Rightarrow$

$\begin{array}{l} x=0 \\ x=-4 \end{array} \Rightarrow \begin{array}{l} (0,0) \\ (-4,0) \end{array}$

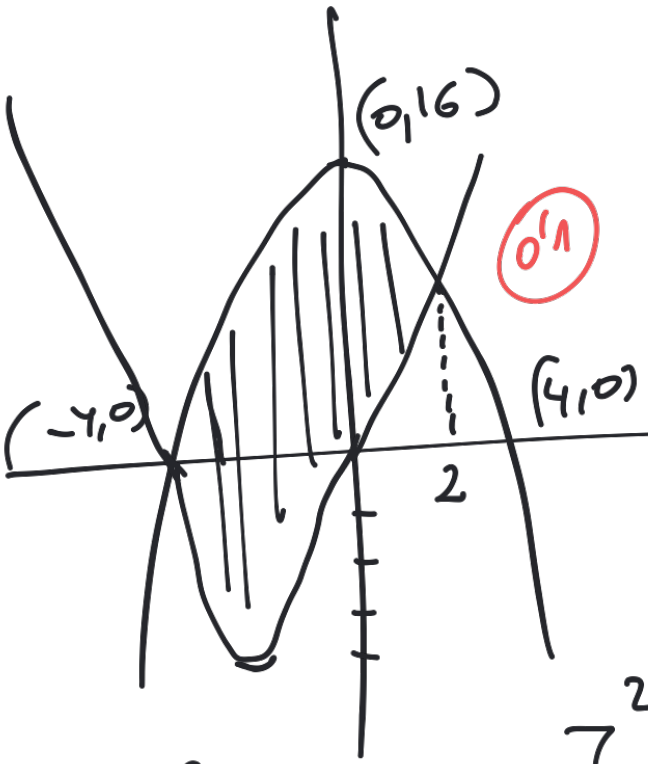
③ $a=1>0, \cup$ CONVEXA

PUNTOS DE CORTE ENTRE $y=f(x)$, $y=g(x) \Rightarrow 16-x^2=x^2+4x$

$2x^2+4x-16=0$

$x^2+2x-8=0$

$x^2+2x-8=0 \Rightarrow x=\frac{-2\pm\sqrt{4+32}}{2}=\frac{-2\pm 6}{2}$ ⟨ $x=-4$, $x=2$ 0'1



ÁREA $=\int_{-4}^{2}\left[(16-x^2)-(x^2+4x)\right]dx=$ 0'2

$$=\int_{-4}^{2}\left[16-x^2-x^2-4x\right]dx=\int_{-4}^{2}(-2x^2-4x+16)\,dx$$

$$=-\frac{2x^3}{3}-\frac{4x^2}{2}+16x\Big]_{-4}^{2}=-\frac{16}{3}-2\cdot 4+16\cdot 2-\left(+\frac{2\cdot 64}{3}-2\cdot 16-64\right)=$$

0'2

$$=-\frac{16}{3}-8+\cancel{32}-\frac{128}{3}+\cancel{32}+64=\frac{-144}{3}+128-8=$$

$$=\frac{-144}{3}+120=-48+120=$$

$=\boxed{72}$ unid² 0'2

c) $\int \frac{-dx}{1+e^x} = \left[\begin{array}{l} e^x = t \Rightarrow \\ e^x dx = dt \\ dx = \frac{dt}{t} \end{array} \right] = \int -\frac{1}{1+t} \cdot \frac{dt}{t} =$

$= \int \frac{-1}{(1+t)t} \, dt = \int \frac{1}{1+t} dt + \int \frac{-1}{t} dt = \ln|1+t| - \ln|t| =$

$\downarrow$

$\frac{-1}{(1+t)t} = \frac{A}{1+t} + \frac{B}{t} \Rightarrow -1 = At + B(1+t)$

$\Rightarrow t = 0 \Rightarrow -1 = B$

$\Rightarrow t = -1 \Rightarrow -1 = -A \Rightarrow A = 1$

$= \ln|1+e^x| - \ln|e^x| + C.$

2) a.i) $\left(\begin{array}{ccc|c} 1 & a & 1 & a+1 \\ -a & 1 & -1 & 2a \\ 0 & -1 & 1 & a \end{array}\right)$ $|A| = 1 + a - 1 + a^2 = a^2 + a$

$|A| = 0 \iff a(a+1) = 0 \Rightarrow a = 0, \; a = -1$ (0'2)

1º) SE $a \neq \{0, -1\} \Rightarrow$ RANGO A = RANGO A* $\Rightarrow$ SISTEMA COMPATIBLE DETERMINADO (0'1) (SOLUCIÓN ÚNICA)

2º) SE $a = 0 \Rightarrow \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix} \Rightarrow$ RANGO A = RANGO A* = 2 SISTEMA COMPATIBLE INDETERMINADO (INFINITAS SOLUCIÓNS) (0'1)

$C_1 = C_3$, $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \neq 0$

3º) SE $a = -1 \Rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & -1 & -2 \\ 0 & -1 & 1 & -1 \end{pmatrix}$ RANGO A = 2 $\neq$ RANGO A* = 3

$\begin{vmatrix} 1 & 1 & 0 \\ 1 & -1 & -2 \\ 0 & 1 & -1 \end{vmatrix} = 1 + 2 + 1 \neq 0$

SISTEMA INCOMPATIBLE (SEN SOLUCIÓN) (0'1)

a.ii) $\left.\begin{array}{l} x + z = 1 \\ y - z = 0 \\ -y + z = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x = 1 - \lambda \\ y = \lambda \\ z = \lambda \end{array}\right\} \Rightarrow$ SOLUCIÓNS $(1-\lambda, \lambda, \lambda), \lambda \in \mathbb{R}$. (0'5)

b) $AM = N^{-1}$

$$\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x & 0 \\ y & 1 \\ x-y & 1 \end{pmatrix} = N^{-1} \Rightarrow AMN = I$$

$$\begin{pmatrix} x+x-y & 1 \\ -x+y & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 2x & 1 \\ -x+y & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} 2x-y-1 & -2x+y+2 \\ -x+y-1 & x-y+2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \left.\begin{array}{l} 2x-y-1 = 1 \\ -2x+y+2 = 0 \\ -x+y-1 = 0 \\ x-y+2 = 1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} 2x-y = 2 \\ -2x+y = -2 \\ -x+y = 1 \\ x-y = -1 \end{array}\right] \rightarrow \begin{array}{r} 2x-y = 2 \\ -x+y = 1 \\ \hline x = 3 \end{array} \quad 0'5 \qquad \begin{array}{l} -x+y = 1 \\ -3+y = 1 \\ \boxed{y = 4} \end{array} \quad 0'5$$

3) r: $\dfrac{x-1}{-1} = y = \dfrac{z}{2} \Rightarrow \begin{array}{l} x-1=-y \Rightarrow \\ 2y=z \Rightarrow \end{array} \boxed{\begin{cases} x+y=1 \\ 2y-z=0 \end{cases}}$ 0'25

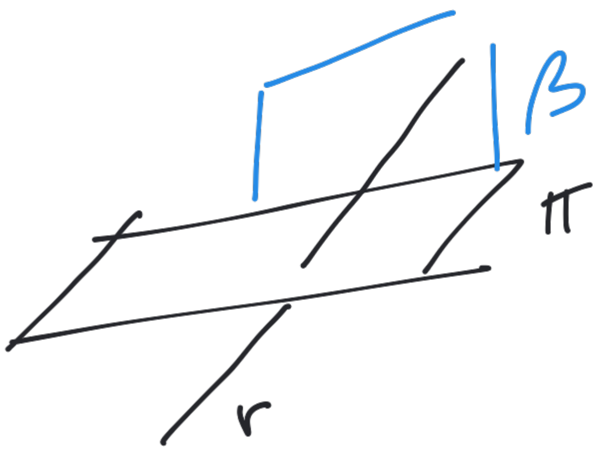
$$\begin{vmatrix} x-1 & 1 & 1 \\ y & 1 & -1 \\ z+1 & 2 & 0 \end{vmatrix} = 2y - z - 1 - z - 1 + 2(x-1) =$$

$$= 2x - 2 + 2y - 2z - 2 = 0$$

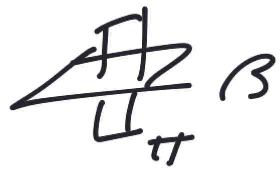
$\Rightarrow 2x + 2y - 2z - 4 = 0 \Rightarrow \boxed{x+y-z-2=0}$ 0'25

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 2 & -1 & 0 \\ 1 & 1 & -1 & 2 \end{array}\right) \rightarrow |M| = -2 - 1 + 1 = -2 \neq 0 \Rightarrow \text{RANGO } M = 3$$

$\Rightarrow$ RANGO M = RANGO M^* $\Rightarrow$ SISTEMA COMPATIBLE DETERMINADO SOLUCIÓN ÚNICA, CÓRTANSE NUN PUNTO. 0'25 b)



$$\beta \begin{cases} \perp \pi \\ r \subset \beta \end{cases} \Rightarrow$$



0'25

$$\Rightarrow \beta \begin{cases} P_r\ (1,0,0) \\ \vec{v}_\pi\ (1,1,-1) \\ v_r\ (-1,1,2) \end{cases} \Rightarrow \vec{v}_N = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ -1 & 1 & 2 \end{vmatrix} = (3,-1,2)$$

0'25

$$\left.\begin{array}{l} 3x - y + 2z + D = 0 \\ 3 + D = 0 \end{array}\right\} \Rightarrow D = -3 \Rightarrow \boxed{3x - y + 2z - 3 = 0}$$

0'25

4)

a) $A(1,2,-3)$, $B(1,5,0)$, $C(5,6,-1)$, $D(4,-1,3)$

$$V = \frac{1}{6} \left| \vec{AB}, \vec{AC}, \vec{AD} \right|$$

$\vec{AB}(0, 3, 3)$
$\vec{AC}(4, 4, 2)$ $\Rightarrow$
$\vec{AD}(3, -3, 6)$ 0'25

$$\left| \vec{AB}, \vec{AC}, \vec{AD} \right| = \begin{vmatrix} 0 & 3 & 3 \\ 4 & 4 & 2 \\ 3 & -3 & 6 \end{vmatrix} = -36 + 18 - 36 - 72$$

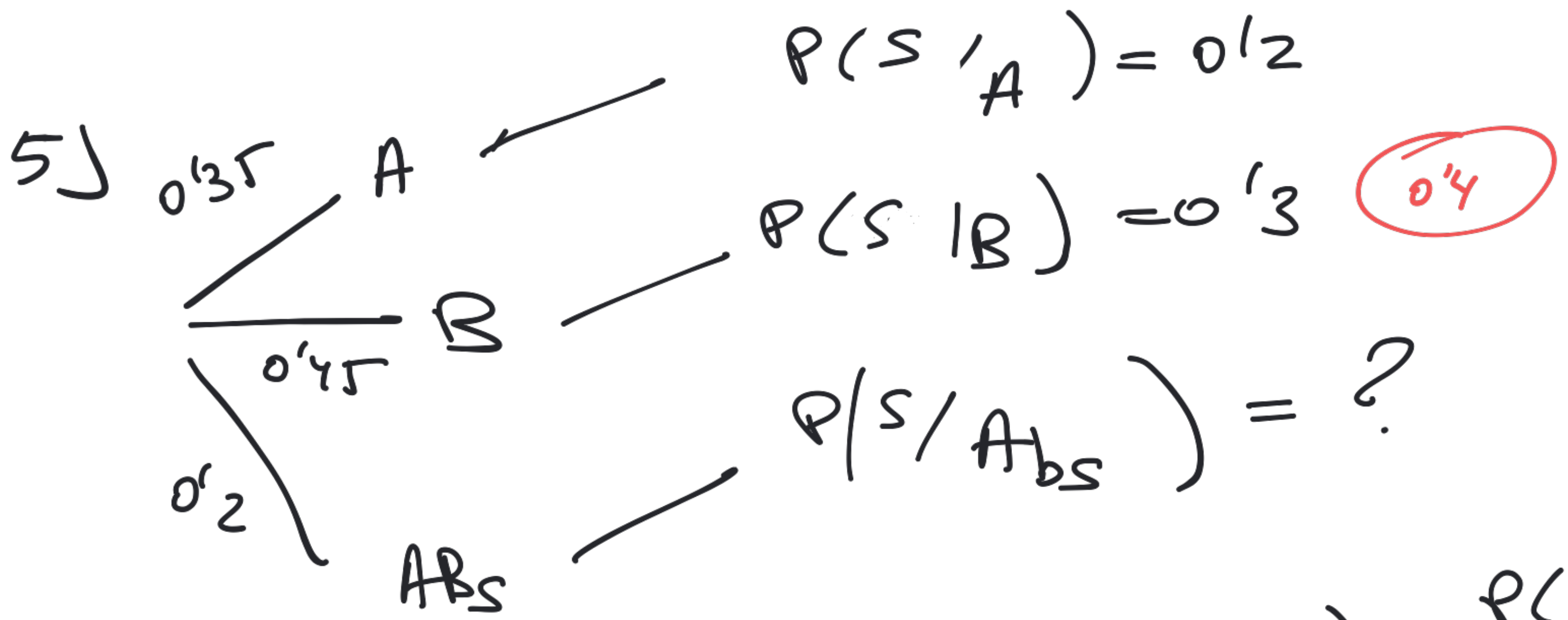
$$= -150 + 18 = -126 \Rightarrow$$ 0'25

$$\Rightarrow V = \frac{1}{6} \left| -126 \right| = \frac{126}{6} = 21 \text{ unid}^3$$ 0'25

b) $$\text{Area} = \frac{1}{2} \left| \vec{AB} \times \vec{AC} \right| = \frac{1}{2} \sqrt{6^2 + 12^2 + 12^2} = \frac{1}{2} \sqrt{36 + 144 + 144} = \frac{18}{2} = 9 \text{ unid}^2$$ 0'25 0'25

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 0 & 3 & 3 \\ 4 & 4 & 2 \end{vmatrix} = (-6, 12, -12)$$ 0'25

5)



$P(S \cap Abs) = 0'03 \Rightarrow P(S/Abs) = \dfrac{P(S \cap Abs)}{P(Abs)} \Rightarrow$

$\Rightarrow P(S/Abs) = \dfrac{0'03}{0'2} = \dfrac{3/100}{2/10} = \dfrac{3 \cdot \cancel{10}}{2 \cdot \cancel{100}} = \dfrac{3}{20} = 0'15$ 0'4

a) $P(S) = 0'35 \cdot 0'2 + 0'45 \cdot 0'3 + 0'2 \cdot 0'15 = 0'235$ 0'2

b) $P(B/\overline{S}) = \dfrac{P(B \cap \overline{S})}{P(\overline{S})} = \dfrac{0'45 \cdot 0'7}{1 - 0'235} = \underline{\underline{0'41176}}$

0'5 0'5