EXAME 3ª AVALIACIÓN 2ºBAC-A

1) a) A FUNCIÓN É CONTINUA EN $R-\{2,3\}$ POR SER CADA FUNCIÓN CONTINUA NO SEU DOMINIO 0'25

EN $x<2$, $f(x)=\frac{3}{x-2}$ CONTINUA EN $R-\{2\} \Rightarrow$ CONTINUA EN $x<2$

EN $2<x<3$, $f(x)=\cos \pi x$ CONTINUA EN $R \Rightarrow$ CONTINUA EN $(2,3)$

EN $x>3$, $f(x)=\frac{\ln(x-2)}{3-x}$ CONTINUA NO SEU DOMINIO, $\left.\begin{matrix} x-2>0 \\ x\neq 3 \end{matrix}\right\} \Rightarrow$

$\Rightarrow$ CONTINUA EN $x>2$, $x\neq 3 \Rightarrow$ CONTINUA EN $x>3$.

ESTUDEMOS A CONTINUIDADE EN $x=2$, $x=3$:

$\boxed{\text{CONTINUIDADE EN } x=2:}$ $\lim_{x\to 2^-} f(x) = \lim_{x\to 2} \frac{3}{x-2} = \frac{3}{0^-} = -\infty$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2} \cos \pi x = \cos 2\pi = 1 \Rightarrow \nexists \lim_{x\to 2} f(x) \Rightarrow$
(calculadora en radianes)

$\Rightarrow$ A FUNCIÓN É DESCONTINUA INEVITABLE DE SALTO INFINITO 0'25

$\boxed{\text{CONTINUIDADE EN } x=3:}$ $\lim_{x\to 3^-} f(x) = \lim_{x\to 3} \cos \pi x = \cos 3\pi = -1$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3} \frac{\ln(x-2)}{3-x} = \left[\frac{\ln(3-2)}{3-3} = \frac{\ln 1}{0} = \frac{0}{0}\right] \underset{L'H}{=}$

$$= \lim_{x \to 3} \frac{\frac{1}{x-2}}{-1} = \lim_{x \to 3} \frac{-1}{x-2} = \frac{-1}{3-2} = -1 \Rightarrow$$

$$\Rightarrow \exists \lim_{x \to 3} f(x) = -1$$

$$f(3) = \cos 3\pi = -1 \Rightarrow \exists \lim_{x \to 3} f(x) = f(3) \Rightarrow$$ A FUNCIÓN É CONTINUA EN $x = 3 \Rightarrow$ A FUNCIÓN É CONTINUA EN $R - \{2\}$.

0'25

$$\text{bii)} \lim_{x \to 1} \left(\frac{2e^{x-1}}{x+1}\right)^{\frac{x}{x-1}} = \left[\left(\frac{2e^{1-1}}{1+1}\right)^{\frac{1}{1-1}} = \left(\frac{2e^0}{2}\right)^{\frac{1}{0}} = 1^{\infty} \right] = L$$

$$\Rightarrow \ln \lim_{x \to 1} \left(\frac{2e^{x-1}}{x+1}\right)^{\frac{x}{x-1}} = \lim_{x \to 1} \ln \left(\frac{2e^{x-1}}{x+1}\right)^{\frac{x}{x-1}} =$$

$$= \lim_{x \to 1} \frac{x}{x-1} \cdot \ln \left(\frac{2e^{x-1}}{x+1}\right) = [\infty \cdot 0] = \lim_{x \to 1} \frac{\ln\left(\frac{2e^{x-1}}{x+1}\right)}{\frac{x-1}{x}} =$$

$$= \left[\frac{0}{0}\right] =$$

$$= \lim_{x \to 1} \frac{\ln\left(\frac{2e^{x-1}}{x+1}\right)}{\frac{x-1}{x}} \underset{L'H}{=} \lim_{x \to 1} \frac{\frac{1}{\frac{2e^{x-1}}{x+1}} \cdot \frac{2e^{x-1}(x+1) - 2e^{x-1}}{(x+1)^2}}{\frac{x - (x-1)}{x^2}} =$$

$$= \lim_{x \to 1} \frac{\frac{1}{2e^{x-1}} \cdot \frac{2e^{x-1}(x+1) - 2e^{x-1}}{x+1}}{\frac{x - x + 1}{x^2}} = \lim_{x \to 1} \frac{\frac{1}{\cancel{2e^{x-1}}} \cdot \frac{\cancel{2e^{x-1}}(x+1-1)}{x+1}}{\frac{1}{x^2}} =$$

$$= \lim_{x \to 1} \frac{\frac{x}{x+1}}{\frac{1}{x^2}} = \lim_{x \to 1} \frac{x \cdot x^2}{x+1} = \frac{1}{2} \Rightarrow$$

$$\ln L = \frac{1}{2} \Rightarrow e^{1/2} = L \Rightarrow \boxed{\lim_{x \to 1} f(x) = \sqrt{e}}$$

b.i) $\lim_{x \to -1} \frac{-e^{x^2-1} - x}{x^2+4x+3} = \left[\frac{-e^{1-1}+1}{1-4+3} = -\frac{1+1}{0} = \frac{0}{0} \right] \underset{L'H}{=}$

$= \lim_{x \to -1} \frac{-e^{x^2-1}(2x) - 1}{2x+4} = \lim_{x \to -1} \frac{-2x e^{x^2-1} - 1}{2x+4} =$

$= \frac{-2(-1) e^{1-1} - 1}{2(-1)+4} = \frac{2e^{0} - 1}{2} = \frac{2-1}{2} = \left. \frac{1}{2} \right]$ 0'5

2) $f(x)=x^6-6x^4$

a) $f'(x)=6x^5-24x^3 \Rightarrow f'(x)=0 \Leftrightarrow 6x^3(x^2-4)=0$

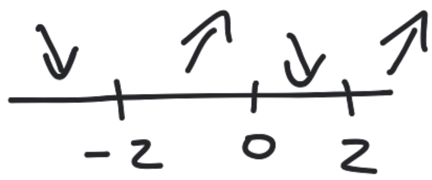
$x=0$ POSIBLES

$x=\pm 2$ EXTREMOS 0'1

$f'(-3)=6(-3)^5-24(-3)^3=-1458+648<0 \Rightarrow$ En $(-\infty,-2)$ A FUNCIÓN É DECRECENTE. 0'1

$f'(-1)=-6+24>0 \Rightarrow (-2,0)$ CRECENTE 0'1

$f'(1)=6-24<0 \Rightarrow (0,2)$ DECRECENTE 0'1

$\Rightarrow$ 

$f'(3)=1458-648>0 \Rightarrow (2,+\infty)$ CRECENTE 0'1

ENTÓN A FUNCIÓN É CRECENTE EN $(-2,0)\cup(2,+\infty)$, DECRECENTE EN $(-\infty,-2)\cup(0,2)$.

MÍNIMO EN $(-2,f(-2))$, $(2,f(2))$

MÁXIMO EN $(0,f(0))$

$f(-2)=(-2)^6-6(-2)^4=64-6\cdot16=64-96=-32 \Rightarrow$

$\Rightarrow (-2,-32)$ mínimo. 0'1

$f(0)=0 \Rightarrow (0,0)$ máximo 0'1

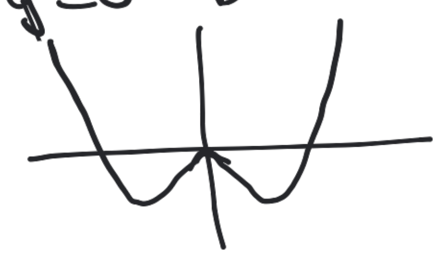
$f(2)=2^6-6\cdot2^4=64-6\cdot16=64-96=-32 \Rightarrow (2,-32)$ mínimo 0'1

ANALICEMOS AS RAMAS E OS PUNTOS DE CORTE COS EIXES COORDENADOS, PARA CONCRETAR SE OS EXTREMOS SON ABSOLUTOS OU RELATIVOS:

$\lim_{x\to\infty} x^6-6x^4=\infty, \quad \lim_{x\to-\infty} x^6-6x^4=\infty$

PUNTOS DE CORTE COS EIXES COORDENADOS: $x=0 \Rightarrow y=0 \Rightarrow (0,0)$

$y=0 \Rightarrow 0=x^6-6x^4 \Rightarrow 0=x^4(x^2-6) \Rightarrow x=0$; $x=\pm\sqrt{6} \Rightarrow (\sqrt{6},0)$, $(-\sqrt{6},0)$



$\Rightarrow$ OS MÍNIMOS SON ABSOLUTOS 0'2
E O MÁXIMO RELATIVO.

c) Área $= 2\int_{-\sqrt{6}}^{0}(-x^6+6x^4)\,dx = \int_{-\sqrt{6}}^{0}(-x^6+6x^4)\,dx + \int_{0}^{\sqrt{6}}(-x^6+6x^4)\,dx =$

0'2 ou

$$\text{Àrea} = 2\cdot\int_{-\sqrt{6}}^{0} (-x^6 + 6x^4)\,dx = 2\left(\underbrace{-\frac{x^7}{7} + \frac{6x^5}{5}}_{0'1}\right]_{-\sqrt{6}}^{0} =$$

$$= -2\left(-\frac{(-\sqrt{6})^7}{7} + \frac{6(-\sqrt{6})^5}{5}\right) =$$

$$= -2\left(\frac{+(\sqrt{6})^7}{7} + \frac{-6(\sqrt{6})^5}{5}\right) = -2\left(\frac{(\sqrt{6})^7}{7} - \frac{6(\sqrt{6})^5}{5}\right) =$$

$$= -2\left(\frac{5(\sqrt{6})^7 - 42(\sqrt{6})^5}{35}\right) = 2\left[\frac{-5(\sqrt{6})^7 + 42(\sqrt{6})^5}{35}\right] =$$

$$= 2\left[\frac{-5\cdot 6^3\sqrt{6} + 42\cdot 6^2\sqrt{6}}{35}\right] = \frac{2}{35}\left[-5\cdot 6^3\sqrt{6} + 42\cdot 6^2\sqrt{6}\right] =$$

$$= \frac{2}{35}\sqrt{6}\left(-5\cdot 6^3 + 42\cdot 6^2\right) = \frac{2\sqrt{6}}{35}\cdot 432 \simeq \underline{\underline{60'46 \text{ unid}}}$$

0'2

3) $A=\begin{pmatrix} m & 1 & 3 \\ 1 & m & 2 \\ 1 & m & 3 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 2 \\ 1 & 0 \\ -1 & 2 \end{pmatrix}$

a)

$|A| = 3m^2 + 3m + 2 - 3m - 2m^2 - 3 = m^2 - 1 \Rightarrow$ 0'2

$|A| = 0 \Leftrightarrow m = \pm 1 \Rightarrow$ 1) Se $m \neq \pm 1 \Rightarrow$ RANGO $A = 3$ 0'2

2°) Se $m = 1 \Rightarrow \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{pmatrix} \Rightarrow \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} \neq 0 \Rightarrow$ RANGO $A = 2$ 0'3

3) Se $m = -1 \Rightarrow \begin{pmatrix} -1 & 1 & 3 \\ 1 & -1 & 2 \\ 1 & -1 & 3 \end{pmatrix} \Rightarrow \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} \neq 0 \Rightarrow$ RANGO $A = 2$ 0'3

b) $A \cdot X = B \Rightarrow$ Se $\left.\begin{array}{l} A \in M_{3\times 3} \\ B \in M_{3\times 2} \end{array}\right\} \Rightarrow X \in M_{3\times 2}$ 0'2

Se $m = 0 \Rightarrow \begin{pmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 1 & 0 & 3 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & 2 \\ 1 & 0 \\ -1 & 2 \end{pmatrix} \Rightarrow \boxed{X = A^{-1} \cdot B}$ 0'2

$A \cdot X = B$

Se $m = 0 \Rightarrow A = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 1 & 0 & 3 \end{pmatrix} \Rightarrow AX = B \Rightarrow \boxed{X = A^{-1}B}$

CALCULEMOS A^{-1}:

$$A = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 1 & 0 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & -1 & 0 \\ -3 & -3 & +1 \\ 2 & +3 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & -3 & 2 \\ -1 & -3 & 3 \\ 0 & 1 & -1 \end{pmatrix}$$

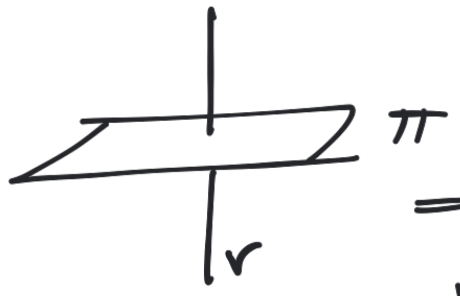
$$|A| = 2 - 3 = -1 \Rightarrow A^{-1} = \begin{pmatrix} 0 & 3 & -2 \\ 1 & 3 & -3 \\ 0 & -1 & 1 \end{pmatrix}$$ 0'3

Comprobación: $\begin{pmatrix} 0 & 3 & -2 \\ 1 & 3 & -3 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 1 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \square$

$$X = \begin{pmatrix} 0 & 3 & -2 \\ 1 & 3 & -3 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 2 \\ 1 & 0 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & -4 \\ 8 & -4 \\ -2 & 2 \end{pmatrix}$$ 0'3

4) a) r: $\frac{x-1}{m} = \frac{y-1}{2} = \frac{z-1}{4}$, π: $x+y+kz=0$ 0'1

a.i) $r \perp \pi \iff \vec{v}_r \parallel \vec{v}_N$ 0'2 $\Rightarrow$ $\vec{v}_r \ (m, 2, 4)$, $\vec{v}_N \ (1, 1, k)$ 0'1 $\Big\} \Rightarrow$



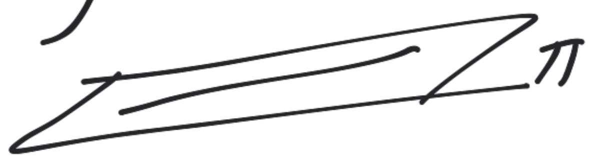
$\Rightarrow \vec{v}_r \parallel \vec{v}_N \iff \frac{m}{1} = \frac{2}{1} = \frac{4}{k}$ 0'2 $\Rightarrow$

$\Rightarrow m = 2$ 0'2

$2k = 4 \Rightarrow k = 2$ 0'2

$\Big\} \Rightarrow r \perp \pi$ se $\boxed{\begin{matrix} m=2 \\ k=2 \end{matrix}}$

a.ii) $r \subset \pi$



$$\left.\begin{matrix} 2x-2 = m(y-1) \\ 4(y-1) = 2(z-1) \end{matrix}\right\} \Rightarrow \begin{matrix} 2x-2 = my-m \\ \boxed{2x - my = -m+2} \\ 4y-4 = 2z-2 \\ \boxed{4y-2z=2} \end{matrix}$$

$$\left(\begin{array}{ccc|c} 2 & -m & 0 & 2-m \\ 0 & 4 & -2 & 2 \\ 1 & 1 & k & 0 \end{array}\right) \Rightarrow |M| = 8k+2m+4$$

$|M| = 0 \Rightarrow \boxed{8k+2m+4=0}$

$m = \frac{-4-8k}{2} = -2-4k$

$m=-2-4k$

$$\begin{pmatrix} 2 & -m & 0 & 2-m \\ 0 & 4 & -2 & 2 \\ 1 & 1 & k & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & k & 0 \\ 0 & 4 & -2 & 2 \\ 2 & -m & 0 & 2-m \end{pmatrix} \to$$

$$\to \begin{pmatrix} 1 & 1 & k & 0 \\ 0 & 2 & -1 & 1 \\ 2 & 2+4k & 0 & 4+4k \end{pmatrix} \to \begin{pmatrix} 1 & 1 & k & 0 \\ 0 & 2 & -1 & 1 \\ 0 & 4k & -2k & 4+4k \end{pmatrix}$$

$\to$ RANGO $M^* =$ RANGO $M \Leftrightarrow$ $4+4k=2k$

$2k=-4$

$\boxed{k=-2}$ 0'5

SE $k=-2 \Rightarrow m=-2-4\cdot(-2)=-2+8=6 \Rightarrow$

$\Rightarrow \boxed{m=6}$ 0'5

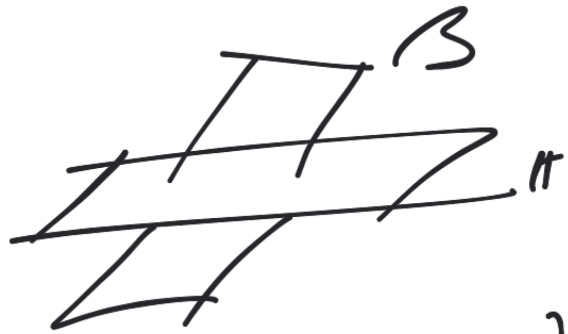
b) $\pi \begin{cases} (0,1,1) \\ (2,0,2) \\ (1,2,6) \end{cases}$ $\beta: \; x-y+z = 3$

$\pi \begin{cases} A(0,1,1) \\ \overrightarrow{AB}(2,-1,1) \\ \overrightarrow{AC}(1,1,5) \end{cases} \Rightarrow \vec{v}_N = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & 1 & 5 \end{vmatrix} = (-6,-9,3) \Rightarrow$

$\Rightarrow \vec{v}_N(-2,-3,1) \Rightarrow \left. \begin{array}{l} -2x-3y+z+D=0 \\ A(0,1,1) \end{array} \right\} \Rightarrow \begin{array}{l} -3+1+D=0 \\ D=2 \end{array}$

$\pi: -2x-3y+z+2=0$ 0'25

$\vec{v}_{N_\beta}(1,-1,1)$, $\vec{v}_{N_\pi}(-2,-3,1) \Rightarrow$



$r \begin{cases} \parallel \pi \\ \parallel \beta \end{cases} \Rightarrow \begin{array}{l} \vec{v}_r \perp \vec{v}_{N_\pi} \\ \vec{v}_r \perp \vec{v}_{N_\beta} \end{array} \Rightarrow \vec{v}_r \parallel \vec{v}_{N_\pi} \times \vec{v}_{N_\beta}$ 0'25 $\Rightarrow r \begin{cases} P(0,0,0) \\ \vec{v}_r(-2,3,5) \end{cases}$ 0'25

$\vec{v}_{N_\pi} \times \vec{v}_{N_\beta} = \begin{vmatrix} i & j & k \\ -2 & -3 & 1 \\ 1 & -1 & 1 \end{vmatrix} = (-2,3,5)$

$r \parallel \pi \Rightarrow$

$P(0,0,0) \notin \pi$, $P(0,0,0) \notin \beta$ 0'25

$\boxed{\text{Válida} \quad \begin{array}{l} -2x-3y+z=0 \\ x-y+z=0 \end{array}}$

5) $P(>30) = 0'2$
$P(D) = 0'08$
$P(>30 \cap D) = 0'06$

$\Rightarrow$ $P(M30) = 0'2$ 0'2
$P(D) = 0'08$ 0'2
$P(M30 \cap D) = 0'06$ 0'2

a) $P(M30 \cap \bar{D}) = ?$ b) $P(\bar{D} \cap \overline{M30})?$ c) $n = 100$, $P(D \cap \overline{M30})$

(a) $P(M30 \cap \bar{D}) = P(M30) - P(M30 \cap D) = 0'2 - 0'06 = 0'14$ 0'4

(b) $P(\bar{D} \cap \overline{M30}) = P(\overline{D \cup M30}) \overset{0'2}{=} 1 - P(D \cup M30) =$
$= 1 - [P(D) + P(M30) - P(D \cap M30)]$ 0'1
$= 1 - [0'08 + 0'2 - 0'06]$ 0'1
$= 1 - [0'28 - 0'06] = 1 - 0'22 = 0'78$

(c) Se $n = 100$, $P(D \cap \overline{M30}) = P(D) - P(D \cap M30) =$
$= 0'08 - 0'06 = 0'02 \Rightarrow 2\%$ 0'3

ENTÓN 2% de 100 $\Rightarrow$ DOS 100 TRABALLADORES 2 SON DIRECTIVOS E NON TEÑEN MÁIS DE 30 ANOS 0'3