(13)

a) $r \begin{cases} A(0,1,3) \\ B(1,1,1) \end{cases} \Rightarrow r \begin{cases} P_r(0,1,3) \\ \vec{v_r} = \vec{AB} = (1,0,-2) \end{cases} \Rightarrow$

$\Rightarrow r: \begin{cases} x = t \\ y = 1 \\ z = 3-2t \end{cases}$

$s: \begin{cases} x+y-2z-1=0 \\ y-2z=0 \end{cases}$ Pasamos s a paramétricas

$\begin{matrix} x+y = 1+2z \\ y = 2z \end{matrix}$ / $\begin{matrix} x+y = 1+2\alpha \\ y = 2\alpha \\ z = \alpha \end{matrix}$ / $\begin{matrix} x = 1 \\ y = 2\alpha \\ z = \alpha \end{matrix}$

$s: \begin{cases} x = 1 \\ y = 2\alpha \\ z = \alpha \end{cases}$ $\begin{cases} P_s(1,0,0) \\ \vec{v_s}(0,2,1) \end{cases}$ $\vec{P_rP_s}(1,-1,-3)$

Como $\vec{v_r}(1,0,-2)$ e $\vec{v_s}(0,2,1)$, non son proporcionais, r e s ou cortante ou cruzante.

$M = \begin{pmatrix} \vec{v_r} \\ \vec{v_s} \end{pmatrix} = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 1 \end{pmatrix}$ $rg\, M = 2$ xa que $\exists \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} \neq 0$

$M^* = \begin{pmatrix} \vec{v_r} \\ \vec{v_s} \\ \vec{P_rP_s} \end{pmatrix} = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 1 \\ 1 & -1 & -3 \end{pmatrix}$ $rg\, M^* = 3$ xa que $\begin{vmatrix} 1 & 0 & -2 \\ 0 & 2 & 1 \\ 1 & -1 & -3 \end{vmatrix} \neq 0$

Polo tanto, como $rg\, M = 2 \neq 3 = rg\, M^* \Rightarrow$ r, s cruzante

b) Plano YZ ten a coordenadas x igual a 0

$\alpha: x=0$. O seu vector normal será

$$\vec{n}\,(1,0,0)$$

$$\left.\begin{array}{l}\vec{v_s}\,(0,2,1)\\ \vec{n}\,(1,0,0)\end{array}\right\} \quad \vec{n}\cdot\vec{v_s} = 0+0+0=0$$

$$\Rightarrow \vec{n} \perp \vec{v_s} \Rightarrow \begin{cases} \text{ou } s \subset \Pi_{xy} \\ \text{ou } s \parallel \Pi_{xy} \end{cases}$$

Collemos un pto cq de s

$P_s(1,0,0)$ e comprobamos se pertence ao

plano. $P_s \in \Pi_{xy}$?

$\Pi_{xy}: x=0$ $\quad \Rightarrow P_s \notin \Pi_{xy} \Rightarrow$

$1=0$? NON!

$\Rightarrow s \not\subset \Pi_{xy}$ Polo tto: $s \parallel \Pi_{xy}$

c) $d(r,\Pi)$

con $\Pi: 2x+z=0 \quad \rightarrow \vec{n_\Pi} = (2,0,1)$

$$r: \begin{cases} x=t \\ y=1 \\ z=3-2t \end{cases} \quad \rightarrow \vec{v_r}\,(1,0,-2)$$

$\vec{v_r}\cdot\vec{n_\Pi} = 2+0+(-2)=0 \Rightarrow \vec{v_r} \perp \vec{n_\Pi} \Rightarrow$

$\Rightarrow r \parallel \Pi$ ou $r \subset \Pi$

Collo pto $P_r \in r$ e comprobo se $P_r \in \Pi$.

$P_r(0,1,3) \in r$, $\quad 2\cdot 0+3 \neq 0 \Rightarrow P_r \notin \Pi \Rightarrow$

$\Rightarrow r \parallel \Pi$. Polo tto, $d(r,\Pi) = \underset{P \in r}{d(P,\Pi)}$

Tomamos $P_r = (0,1,3) \in r$

$$d(r,\pi) = d(P_r,\pi) = \frac{|2.0 + 1.0 + 3.1|}{\sqrt{2^2+1^2}} = \frac{3}{\sqrt{5}} =$$

$\vec{n_\pi}\ (2,0,1)$

$$= \frac{3\sqrt{5}}{5}\ u$$

(14)

$\alpha: 2x - 2y + 4z - 7 = 0$

$\beta: \begin{cases} x = 1 - \lambda + 3\mu \\ y = 5 + \lambda + \mu \\ z = 4 + \lambda - \mu \end{cases}$ $\quad r: \begin{cases} x + 2z - 3 = 0 \\ y - 5 = 0 \end{cases}$

a) Pos relativa de α e β.

Calculo ec xeral de β

$$\begin{vmatrix} x-1 & y-5 & z-4 \\ -1 & 1 & 1 \\ 3 & 1 & -1 \end{vmatrix} = 0 \Rightarrow \beta: 2x - 2y + 4z - 8 = 0$$

ou $\beta: x - y + 2z - 4 = 0$

Temos que $\vec{n_\alpha} = (2,-2,4)$
$\vec{n_\beta} = (1,-1,2)$

Os vectores son proporcionais, os planos van ser paralelos ou coincidentes

$$\frac{2}{1} = \frac{-2}{-1} = \frac{4}{2} \neq \frac{-7}{-4} \Rightarrow \alpha \parallel \beta.$$

$$d(\alpha,\beta) = \underset{P\in\alpha}{d(P,\beta)} = \underset{Q\in\beta}{d(Q,\alpha)}$$

Collo $Q \in \beta$, $Q(1,5,4)$, $d(\alpha,\beta) = d(Q,\alpha) =$

$$= \frac{|2.1 - 2.5 + 4.4 - 7|}{\sqrt{2^2 + (-2)^2 + 4^2}} = \frac{1}{\sqrt{24}} = \frac{\sqrt{24}}{24}\ u$$

b) Calcule Ec xeral do plano $\pi \perp \alpha$, $\pi \supset r$

$\pi \begin{cases} P_r\ (3,5,0) \\ \vec{n_\alpha}\ (2,-2,4) \\ \vec{v_r}\ (-2,0,1) \end{cases}$ (*) $\leftarrow$ π está determinado por un pto de r, o vector director de r, e o vector $\vec{n_\alpha}$ que está contido no plano pq $\alpha \perp \pi$.

$r: \begin{cases} x+2z-3=0 \\ y-5=0 \end{cases}$

Paso r a paramétricas

$\begin{cases} x = 3-2\alpha \\ y=5 \\ z=\alpha \end{cases}$

(*) $\begin{cases} P_r\ (3,5,0) \\ \vec{v_r}\ (-2,0,1) \end{cases}$

Então

$$\pi: \begin{vmatrix} x-3 & y-5 & z \\ 2 & -2 & 4 \\ -2 & 0 & 1 \end{vmatrix} = 0$$

$$\pi: x+5y+2z-28=0$$

c)

$P: r \cap XY \rightarrow \begin{cases} z=0 \\ x+2z-3=0 \\ y-5=0 \end{cases} \rightarrow P(3,5,0)$

$\pi_{XY}: z=0$

$Q: r \cap YZ \rightarrow \begin{cases} x=0 \\ x+2z-3=0 \\ y-5=0 \end{cases} \rightarrow Q(0,5,3/2)$

$\pi_{YZ}: x=0$

$$d(P,Q) = |\vec{PQ}| = \sqrt{(-3)^2+0^2+\left(\frac{3}{2}\right)^2} = \frac{3\sqrt{5}}{2}\ u$$