1) Sol. Ficha ABAU (P-1)
Probabilidade

Muestra 250 pacientes: M: $\frac{150}{250}=\frac{3}{5}$ → $P(C/M)=0'8$; $P(\overline{C}/M)=0'2$
$\overline{M}$; $\frac{100}{250}=\frac{2}{5}$

M = "toma medicamento" ; $P(M)=\frac{3}{5}$; C = "se cura"
$\overline{M}$ = "toma placebo" ; $P(\overline{M})=\frac{2}{5}$; $\overline{C}$ = "No se cura"

P(de que se cure si ha tomado el medicamento) = 0'8
$= P(C/M)$

$\mathcal{E}$ = "elegir un paciente al azar de la muestra y observar los resultados"

P(tomase placebo o no se curase) $= P(\overline{M}\cup\overline{C}) \underset{\text{Morgan}}{=} P(\overline{M\cap C})$
$= 1 - P(M\cap C)$?

Para calcularlo tengo en cuenta que $P(C/M) =: \frac{P(C\cap M)}{P(M)} = 0'8$
despejando $P(C\cap M) = 0'8\cdot P(M) = 0'8\cdot\frac{3}{5} = \boxed{0'48}$

Ahora sustituyo en $P(\overline{M}\cup\overline{C}) = 1 - P(M\cap C) = 1 - 0'48 = \boxed{0'52}$
Solución

2) En una comarca, los habitantes cultivan en sus jardines, entre otras cosas, camelias y rosas
$\mathcal{E}$ = "extraer 1 habitante al azar y observar sus cultivos"
Se observa que:
- 40% cultivan Camelias ⇒ $P(C)=0'4$
- 35% cultivan Rosas ⇒ $P(R)=0'35$
- 21% cultivan Camelias y Rosas ⇒ $P(C\cap R)=0'21$

a) $P(C\cup R)$ = ?
$P(C\cup R) = P(C) + P(R) - P(C\cap R)$
$P(C\cup R) = 0'4 + 0'35 - 0'21 = \boxed{0'54}$

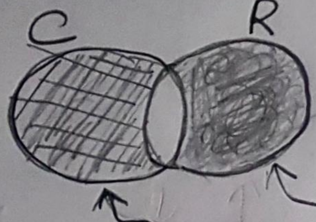
b) $P(\overline{C}\cap\overline{R})$ = ? por las leyes de Morgan sabemos:
$P(\overline{C}\cap\overline{R}) = P(\overline{C\cup R}) = 1 - P(C\cup R) =$
$= 1 - 0'54 = \boxed{0'46}$

② continuación

c) $P(C/R) = \dfrac{P(C\cap R)}{P(R)} = \dfrac{0'21}{0'35} = \boxed{0'6}$

d) $P(R/C) = \dfrac{P(R\cap C)}{P(C)} = \dfrac{0'21}{0'4} = \boxed{0'525}$

e)



$\left\{ \begin{array}{l} R\cap\overline{C} = \text{cultiva Rosas y No camelias} \\ C\cap\overline{R} = \text{cultiva Camelias y No Rosas} \end{array} \right.$

son dos sucesos incompatibles

* por tanto $(C\cap\overline{R}) \cup (R\cap\overline{C})$ (es lo mismo que $(C-R)\cup(R-C)$)

es una unión disjunta

* por ser disjunta $\Rightarrow$ $P[(C\cap\overline{R})\cup(R\cap\overline{C})] = P(C\cap\overline{R}) + P(R\cap\overline{C})$

~ calculamos: $P(C\cap\overline{R}) = P(C) - P(C\cap R) = 0'4 - 0'21 = \boxed{0'19}$

mirar dibujo

~ calculamos: $P(R\cap\overline{C}) = P(R) - P(R\cap C) = 0'35 - 0'21 = \boxed{0'14}$

$\Rightarrow P[(C\cap\overline{R})\cup(R\cap\overline{C})] = 0'19 + 0'14 = \boxed{0'33}$ sólo rosas ó sólo camelias

③ $A, B \in \Omega$ t.q. $P(B) = 0'8$, $P(A\cap B) = 0'2$, $P(A\cup B) = 3P(A)$

Calcula $P(A)$?

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$3P(A) = P(A) + 0'8 - 0'2$

$2P(A) = 0'6$

$\boxed{P(A) = 0'3}$

④ R = "se acuerda y la riega" ; $P(R) = \frac{2}{3}$

$\overline{R}$ = "no se acuerda y no la riega". $P(\overline{R}) = \frac{1}{3}$

S = "sobrevive"

$\overline{S}$ = "no sobrevive"

$P(S)$? no la conocemos

ponemos nombres

* Apuntamos los datos:

♀
- $P(R)=\frac{2}{3}$ — $P(S/R)=0'7$
  - $P(\overline{S}/R)=0'3$
- $P(\overline{R})=\frac{1}{3}$ — $P(S/\overline{R})=0'2$
  - $P(\overline{S}/\overline{R})=0'8$

No hacen falta

Al final el rosal S = "sobrevive"

Nos piden $\boxed{P(\overline{R}/S)} = \dfrac{P(\overline{R}\cap S)}{P(S)}$?

* Calculamos $P(S) = \underbrace{P(S\cap R)+P(S\cap\overline{R})}_{\text{sucesos incompatibles unión disjunta}}$

~ usando los datos que tenemos:

$$0'7=P(S/R)=\frac{P(S\cap R)}{P(R)}=\frac{P(S\cap R)}{\frac{3}{2}} \Rightarrow P(S\cap R)=0'7\cdot\frac{2}{3}\simeq\boxed{0'4\widehat{6}}$$

$$0'2=P(S/\overline{R})=\frac{P(S\cap\overline{R})}{P(\overline{R})}=\frac{P(S\cap\overline{R})}{\frac{1}{3}} \Rightarrow P(S\cap\overline{R})=0'2\cdot\frac{1}{3}\simeq\boxed{0'0\widehat{6}}$$

$$\Rightarrow P(S)=P(S\cap R)+P(S\cap\overline{R})\simeq\boxed{0'5\widehat{3}}$$

* Calculamos $P(\overline{R}\cap S)=P(S\cap\overline{R}) \underset{\text{visto antes}}{=} 0'2\cdot\frac{1}{3}\simeq\boxed{0'0\widehat{6}}$

$$\Rightarrow P(\overline{R}/S)=\frac{P(\overline{R}\cap S)}{P(S)}\simeq\frac{0'0\widehat{6}}{0'5\widehat{3}}\simeq\boxed{0'125}$$

⑤ $P(A)=0,2$; $P(B)=0,4$; $P(A\cup B)=0'5$

a) $P(\overline{A})=1-P(A)=1-0'2=\boxed{0'8}$

b) $P(\overline{B})=1-P(B)=1-0'4=\boxed{0'6}$

c) $P(A\cap B)=P(A)+P(B)-P(A\cup B)=0'2+0'4-0'5=\boxed{0'1}$

d) $P(\overline{A}\cup\overline{B}) \underset{\text{Morgan}}{=} P(\overline{A\cap B})=1-P(A\cap B)=1-0'1=\boxed{0'9}$

e) $P(A)\cdot P(B)=0'2\cdot 0'4=0'08\neq P(A\cap B)=0'1 \Rightarrow$ NO son indep.

⑥ 40% Camelias; 35% Rosas; 21% Camelias y Rosas

a) $P(\overline{\overline{R}\cap\overline{C}})$

b) $P(R/C)$

c) $P(R\cap\overline{C})$

d) $P(C\cap\overline{R})$

(6) $P(R) = 0'35$, $P(C) = 0'4$, $P(C\cap R) = 0'21$

a) $P(\overline{R}\cap\overline{C}) \underset{\text{Morgan}}{=} P(\overline{R\cup C}) = 1 - P(R\cup C) = $ (P. 4)

$P(R\cup C) = P(R) + P(C) - P(R\cap C) = 0'35 + 0'4 - 0'21$

$P(R\cup C) = \boxed{0'54} \Rightarrow P(\overline{R}\cap\overline{C}) = 1 - P(R\cup C) = 1 - 0'54 = \boxed{0'46}$

b) $P(R/C) = \dfrac{P(R\cap C)}{P(C)} = \dfrac{0'21}{0'4} = \boxed{0'525}$

c) $P(R\cap\overline{C}) = P(R) - P(R\cap C) = 0'35 - 0'21 = \boxed{0'14}$

d) $P(C\cap\overline{R}) = P(C) - P(R\cap C) = 0'4 - 0'21 = \boxed{0'19}$

$P(\underbrace{(R\cap\overline{C})\cup(C\cap\overline{R})}) = 0'14 + 0'19 = \boxed{0'33}$

unión disjunta

sucesos incompatibles

(7) $P(A) = \frac{2}{5}$, $P(\overline{A}\cup\overline{B}) = \frac{14}{15}$, $P(A\cup B) = \frac{2}{3}$

a) i) $P(B) = ?$

i) $P(\overline{A}\cup\overline{B}) \underset{\text{Morgan}}{=} P(\overline{A\cap B}) = 1 - P(A\cap B) = \frac{14}{15}$

$\Rightarrow P(A\cap B) = 1 - \frac{14}{15} = \boxed{\frac{1}{15}}$

$P(A\cap B) = P(A) + P(B) - P(A\cup B)$

$\frac{1}{15} = \frac{2}{5} + P(B) - \frac{2}{3} \Rightarrow P(B) = \frac{1}{15} - \frac{2}{5} + \frac{2}{3} = \boxed{\frac{1}{3}}$

ii) $P(A\cap\overline{B}) = P(A) - P(A\cap B) = \frac{2}{5} - \frac{1}{15} = \frac{5}{15} = \boxed{\frac{1}{3}}$

b) $P(B/A) = \dfrac{P(B\cap A)}{P(A)} = \dfrac{1/15}{2/5} = \dfrac{5}{30} = \boxed{\frac{1}{6}} \neq \frac{1}{3}$ No son indep.

(8)

Ana: 40% de los días, $P(A) = 0'4$ → $P(T/A) = 0'05$, $P(\overline{T}/A) = 0'95$

Luis: 60% de los días, $P(L) = 0'6$ → $P(T/L) = 0'08$, $P(\overline{T}/L) = 0'92$

No hace falta

$P(T)?$ $T = \underbrace{(T\cap A)\cup(T\cap L)}_{\text{disjunta}}$; $P(T) = P(T\cap A) + P(T\cap L)$

$0'05 = P(T/A) = \dfrac{P(T\cap A)}{P(A)} = \dfrac{P(T\cap A)}{0'4} \Rightarrow \boxed{P(T\cap A)} = 0'4\cdot 0'05 = \boxed{0'02}$

$0'08 = P(T/L) = \dfrac{P(T\cap L)}{P(L)} = \dfrac{P(T\cap L)}{0'6} \Rightarrow P(T\cap L) = \boxed{0'048}$

continuación (8) (P.5)

a) $P(T) = P(T \cap A) + P(T \cap L) = 0'02 + 0'048 = \boxed{0'068}$

b) $P(A/T) = \dfrac{P(A \cap T)}{P(T)} = \dfrac{0'02}{0'068} \simeq \boxed{0'2941}$

(9)

200 bufandas marca A —— $P(D/A) = 0.01$
150 " " " " B —— $P(D/B) = 0.02$
50 " " " " C —— $P(D/C) = 0.04$

400 bufandas

$P(A) = \dfrac{200}{400} = \dfrac{1}{2} = 0'5$; $P(B) = \dfrac{150}{400} = 0,375$; $P(C) = \dfrac{50}{400} = 0,125$

a) $P(A \cup D) = ?$

$$\begin{cases} P(D/A) = 0,01 = \dfrac{P(D \cap A)}{P(A)} = \dfrac{P(D \cap A)}{0'5} \Rightarrow \boxed{P(D \cap A)} = 0'01 \cdot 0'5 = \boxed{0'005} \\ P(D/B) = 0,02 = \dfrac{P(D \cap B)}{P(B)} = \dfrac{P(D \cap B)}{0,375} \Rightarrow \boxed{P(D \cap B)} = 0'02 \cdot 0'375 = \boxed{0'0075} \\ P(D/C) = 0,04 = \dfrac{P(D \cap C)}{P(C)} = \dfrac{P(D \cap C)}{0,125} \Rightarrow P(D \cap C) = 0,04 \cdot 0,125 = \boxed{0'005} \end{cases}$$

$P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C) =$
$= 0'005 + 0'0075 + 0'005 = \boxed{0'0175}$

$P(A \cup D) = P(A) + P(D) - P(A \cap D) = 0'5 + 0'0175 - 0'005 = \boxed{0'5125}$

b) $P(\overline{D} \cap \overline{C}) \underset{\substack{\uparrow \\ \text{Morgan}}}{=} P(\overline{D \cup C}) = 1 - P(D \cup C) = 1 - [P(D) + P(C) - P(D \cap C)]$

$= 1 - [0'0175 + 0,125 - 0,04 \cdot 0,125] = 1 - 0'1375 = \boxed{0'8625}$

c) $P(B/\overline{D}) = \dfrac{P(B \cap \overline{D})}{P(\overline{D})} = \dfrac{P(B) - P(B \cap D)}{1 - P(D)} = \dfrac{0.375 - 0'0075}{1 - 0'0175}$

$= \dfrac{0'3675}{0'9825} \simeq \boxed{0'374}$

P.6

10 1 fábrica, 3 máquinas

$$\begin{cases} A \text{ — } P(D/A) = 0'02 \\ B \text{ — } P(D|B) = 0'04 \\ C \text{ — } P(D|C) = 0'05 \end{cases}$$

$P(A) = P(B) = P(C) = \frac{1}{3}$

a) $P(D) \overset{\text{prob. Totales}}{=} P(A) \cdot P(D/A) + P(B) \cdot P(D/B) + P(C)\, P(C/B)$

$= \frac{1}{3} \cdot 0'02 + \frac{1}{3} \cdot 0'04 + \frac{1}{3}\, 0'05 = \frac{1}{3} \cdot 0'11 \simeq \boxed{0'0367}$

b) $P(A|\overline{D}) \underset{\text{Bayes}}{=} \dfrac{P(A\cap \overline{D})}{P(\overline{D})} = \dfrac{P(A-D)}{P(\overline{D})} = \dfrac{P(A) - P(A\cap D)}{1 - P(D)} = \dfrac{\frac{1}{3} - P(A\cap D)}{1 - 0'0367}$

* calculo $P(A\cap D)$; como $P(D/A) \underset{\text{Bayes}}{=} \dfrac{P(D\cap A)}{P(A)} = 0'02$

$0'02 = \dfrac{P(A\cap D)}{1/3} \Rightarrow P(A\cap C) = \frac{1}{3} \cdot 0'02$

* sustituyo en lo de antes: $P(A|\overline{D}) = \dfrac{1/3 - \frac{1}{3} \cdot 0'02}{1 - 0'0367} =$

$\simeq \dfrac{\frac{1}{3}\, 0'98}{0'9633} \simeq \dfrac{0'3266}{0'9633} \simeq \boxed{0'3391}$

11 a) $P(\overline{A}) = 0'4$
$P(B) = 0'7$
A, B indep.

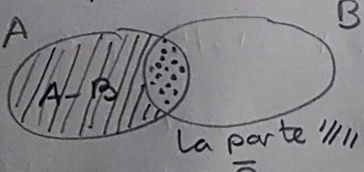
i) $P(A\cup B) = P(A) + P(B) - P(A\cap B) =$
$\underset{\text{A,B indep.}}{=} P(A) + P(B) - P(A) \cdot P(B) =$

$= 0'6 + 0'7 - 0'6 \cdot 0'7 = \boxed{0'88}$

ii) $P(A - B) = P(A\cap \overline{B}) = P(A) - P(A\cap B) \underset{\text{A,B indep.}}{=} 0'6 - 0'6 \cdot 0'7 =$

$= \boxed{0'18}$

¡ojo! esta propiedad (que usamos muchas veces) al parecer hay que demostrarla en la ABAU aunque no lo pidan



La parte //// es A-B

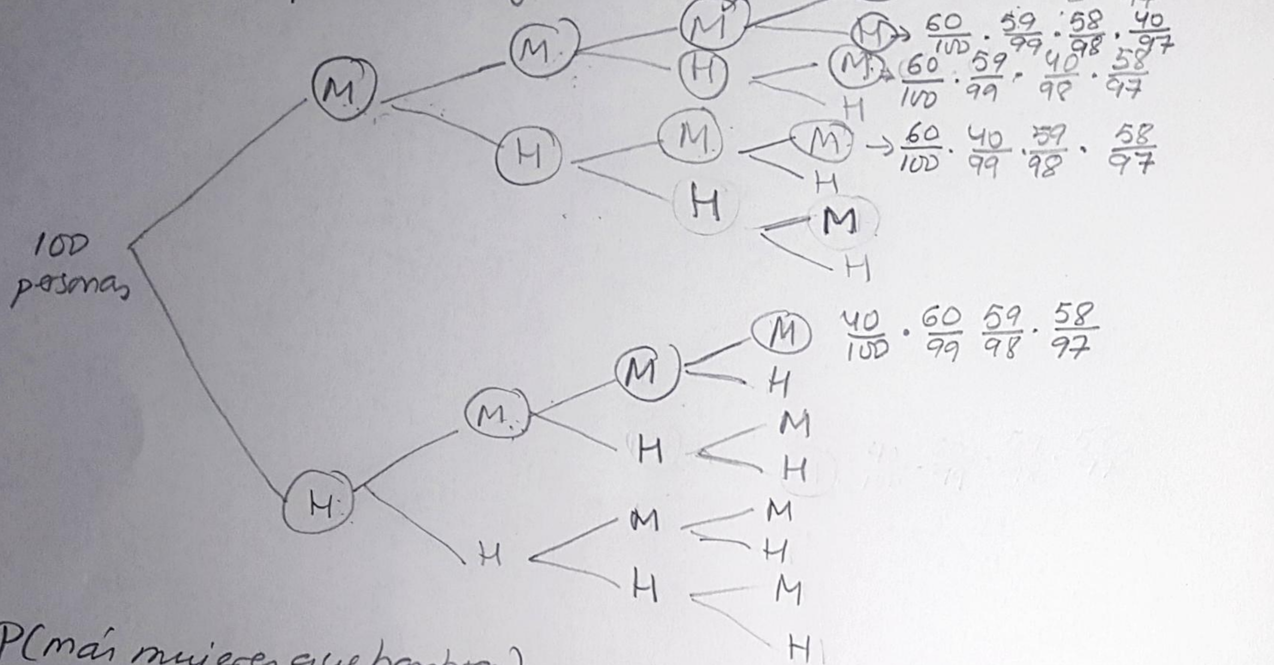
$A - B = A\cap \overline{B}$

$A = \underbrace{(A-B) \cup (A\cap B)}_{\text{unión disjunta (ver dibujo)}} \Rightarrow P(A) = P(A-B) + P(A\cap B) \Rightarrow$

$\Rightarrow \underline{P(A-B) = P(A) - P(A\cap B)}$

11) b) H = la persona elegida es hombre p.7

M = la persona elegida es mujer



P(más mujeres que hombres) =

$$\underbrace{\frac{60}{100}\cdot\frac{59}{99}\cdot\frac{58}{98}\cdot\frac{57}{97}}_{4\text{ mujeres}}+4\underbrace{\left(\frac{60\cdot 59\cdot 58\cdot 40}{100\cdot 99\cdot 98\cdot 97}\right)}_{3\text{ mujeres}}=\frac{11703240+32851200}{100\cdot 99\cdot 98\cdot 97}=$$

$$=\frac{44{,}554.440}{94\,109\,400}=0'473432409\simeq\boxed{0'4734}$$

——— o ———

(12)

$P(F) = 0'3$

$P(H/F) = 0'6$

$P(M/\bar{F}) = 0'7$

$\mathcal{E}$ = "elegir un paciente al azar"

a) $P(M) = P(M\cap F) + P(M\cap\bar{F}) = 0'49 + 0'12 = \boxed{0'61}$

$$0'7 = P(M/\bar{F}) = \frac{P(M\cap\bar{F})}{P(\bar{F})} = \frac{P(M\cap\bar{F})}{1-P(F)} = \frac{P(M\cap\bar{F})}{0'7}$$

$\boxed{0'49 = P(M\cap\bar{F})}$

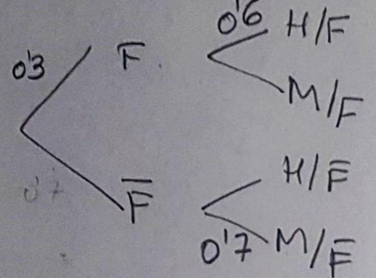
$0'3 = P(F) = P(F\cap M) + P(F\cap H) = P(F\cap M) + 0'18 \Rightarrow P(F\cap M) = 0'3 - 0'18 = 0'12$

$$0'6 = P(H/F) = \frac{P(H\cap F)}{P(F)} = \frac{P(H\cap F)}{0'3} \Rightarrow P(H\cap F) = 0'6\cdot 0'3 = 0'18$$

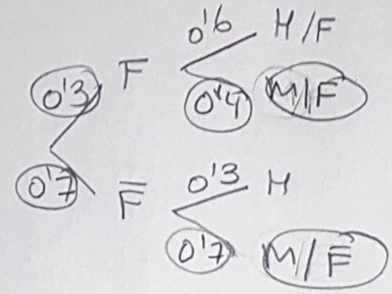
b) $$P(F/H) = \frac{P(F\cap H)}{P(H)} = \frac{P(F\cap H)}{1-P(M)} = \frac{0'18}{1-0'61} = 0'461538461 \simeq$$

$\simeq \boxed{0'4615}$

12 De otra forma:



Completando el esquema



Completamos cuadro

Datos

a) $P(M) = P(M/F)\cdot P(F) + P(M/\overline{F})\cdot P(\overline{F}) =$

$= 0'4 \cdot 0'3 + 0'7 \cdot 0'7 = \boxed{0'61}$

b) $P(F/H) = \dfrac{P(F\cap H)}{P(H)} = \dfrac{0'6\cdot 0'3}{1-0'61} = \dfrac{0'18}{0'39} \simeq 0'4615$

$\left[0'6 = P(H/F) = \dfrac{P(F\cap H)}{P(F)} = \dfrac{P(F\cap H)}{0'3} \Rightarrow P(F\cap H) = 0'6\cdot 0'3 \right]$

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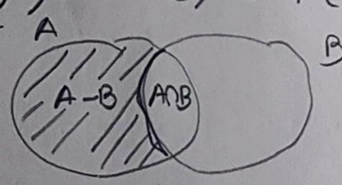
13 $\left.\begin{array}{l} P(A)=0{,}7 \\ P(B)=0{,}6 \\ P(A\cup B)=0{,}9 \end{array}\right\}$ a) indep? $P(A\cap B) = P(A)+P(B)-P(A\cup B)$

$P(A\cap B) = 0'7 + 0'6 - 0'9 = \boxed{0'4}$

$P(A)\,P(B) = 0'7\cdot 0'6 = \boxed{0'42}$

NO son indep.

b) i) $P(A-B) = P(A\cap\overline{B}) = P(A) - P(A\cap B) = 0'7 - 0'4 = \boxed{0'3}$



$P(A) = P(A-B) + P(A\cap B)$

despejando:

$P(A-B) = P(A) - P(A\cap B)$

$A = (A-B)\cup(A\cap B)$

unión disjunta

b ii) $P(A/\overline{B}) = \dfrac{P(A\cap\overline{B})}{P(\overline{B})} = \dfrac{0'3}{0'4} = \boxed{0'75}$

Bayes