6

Number and algebra

Fractions and percentages

A percentage is a special type of fraction whose denominator is 100, for example, $95\% = \frac{95}{100}$. The symbol % is an abbreviation of '/100' and 'per cent', which comes from the Latin *per centum*, meaning 'by the hundred'. Percentages are all around us: with interest earned on savings, discounts on purchases, profits for businesses, opinion polls, sports statistics, school reports, … Percentages are a useful way of making comparisons. An understanding of percentages is vital.

Chapter outline

	Proficiency strands				
6-01 Fractions	U	F			
6-02 Adding and subtracting fractions	U	F	PS		
6-03 Multiplying and dividing fractions	U	F	PS		
6-04 Percentages, fractions and decimals	U	F			C
6-05 Fraction and percentage of a quantity	U	F			C
6-06 Expressing amounts as fractions and percentages	U	F			C
6-07 Percentage increase and decrease	U	F	PS		C
6-08 Percentages without calculators	U	F	PS	R	
6-09 The unitary method	U	F	PS	R	C
6-10 Profit, loss and GST	U	F	PS	R	C
6-11 Percentage problems	U	F	PS		C

Wordbank

cost price The price an item costs the retailer

discount The saving made between the original price of an item and the reduced price

improper fraction A fraction such as $\frac{7}{3}$, in which the numerator is larger than or equal to the denominator.

loss The amount lost when selling an item at a lower price

GST Goods and services tax, a 10% tax added to the original price of an item or service

profit The amount made when selling an item at a higher price

selling price The price at which an item is sold by the retailer

unitary method A method for finding the whole amount when a percentage of that amount is known, by first finding the size of 1%

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In this chapter you will:

- compare fractions using equivalence
- solve problems involving addition and subtraction of fractions, including those with unrelated denominators
- multiply and divide fractions using efficient written strategies and digital technologies
- connect fractions, decimals and percentages and carry out simple conversions
- find fractions and percentages of quantities and express one quantity as a fraction or percentage of another, with and without digital technologies
- solve problems involving the use of percentages, including percentage increases, percentage decreases and the unitary method, with and without digital technologies
- solve problems involving profit, loss, and goods and services tax (GST), with and without digital technologies

SkillCheck

Worksheet
StartUp assignment 6
MAT08NAWK10049

1 Copy and complete each pair of equivalent fractions.

a $\frac{1}{2} = \frac{}{6}$ **b** $\frac{2}{3} = \frac{}{12}$ **c** $\frac{4}{5} = \frac{16}{}$ **d** $\frac{7}{28} = \frac{1}{}$

e $\frac{3}{4} = \frac{}{8}$ **f** $\frac{15}{20} = \frac{}{4}$ **g** $\frac{6}{10} = \frac{24}{}$ **h** $\frac{3}{5} = \frac{9}{}$

2 Write each percentage as a fraction.

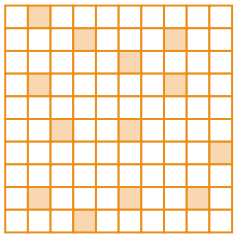
a 17% **b** 3% **c** 51% **d** 63% **e** 89%

3 Evaluate each difference.

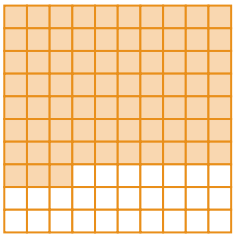
a 100% − 30% **b** 100% − 95% **c** 100% − 39%
d 100% − 71% **e** 100% − 12% **f** 100% − 7%

4 Write the percentage that is shaded in each diagram.

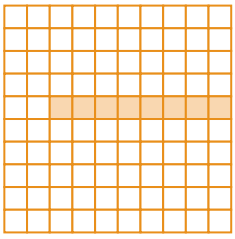
a



b



c



5 Write the percentage that is unshaded in each diagram in question **4**.

6 On a plane, 10% of the seats are first-class and 25% are business class. The rest are economy class. What percentage of the seating is economy class?

7 Write all of the factors of 100.

8 Evaluate each expression.

a 50% of $80 **b** 50% of 12 kg **c** 25% of $20
d 25% of 40 kg **e** 10% of $30 **f** 10% of 20 kg

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6-01 Fractions

Worksheet
Fractions wall
MAT08NAWK10050

Skillsheet
Fractions
MAT08NASS10024

Skillsheet
Improper fractions and mixed numerals
MAT08NASS10025

Animated example
Rational numbers
MAT08NAAE00007

Maths clip
Working out fractions
MAT08NAMC00003

Skillsheet
Equivalent fractions
MAT08NASS10026

Scientific calculators have a fraction key for entering fractions: $\square\!\!\square$ or $a^b/_c$
Some calculators have two ways of entering fractions: MATH mode or LINE mode. MATH mode allows you to enter the numerator and denominator into two blank spaces on the calculator's screen, while LINE mode makes the fraction key act like a vinculum (fraction bar). LINE mode is used in the following examples.

Example 1

Convert $\frac{27}{4}$ into a mixed numeral.

Solution

$\frac{27}{4} = 27 \div 4$
$= 6$ remainder 3
$= 6\frac{3}{4}$

To find the number of 'wholes' in 27 quarters, divide 27 by 4.

Write the remainder in the numerator of a fraction.

Or on a calculator, enter: 27 $a^b/_c$ 4 =.

On a calculator, a mixed numeral can be converted to an improper fraction using the improper fraction key: d/c or $\square\!\!\square$ (this may require the SHIFT or 2ndF key).

Example 2

Convert $4\frac{2}{5}$ to an improper fraction.

Solution

$4\frac{2}{5} = \frac{5 \times 4 + 2}{5} = \frac{22}{5}$

This works because $4\frac{2}{5} = 4 + \frac{2}{5} = \frac{5 \times 4}{5} + \frac{2}{5}$

Or on a calculator, enter: 4 $a^b/_c$ 2 $a^b/_c$ 5 = d/c.

Example 3

Which fraction is larger, $\frac{4}{10}$ or $\frac{3}{8}$?

Solution

Method 1

Convert both fractions so that they share a common denominator of $10 \times 8 = 80$.

$\frac{4}{10} = \frac{4 \times 8}{10 \times 8} = \frac{32}{80}$ and $\frac{3}{8} = \frac{3 \times 10}{8 \times 10} = \frac{30}{80}$

Note that we multiply the numerator and denominator of each fraction by the denominator of the **other** fraction

By comparing numerators, $\frac{32}{80} > \frac{30}{80}$

$\therefore \frac{4}{10}$ is larger.

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Method 2
Another suitable denominator is the lowest common denominator (LCD) of both denominators.
The LCD of 10 and 8 is 40.
Convert both fractions.
$\frac{4}{10} = \frac{4 \times 4}{10 \times 4} = \frac{16}{40}$ and $\frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$
By comparing numerators, $\frac{16}{40} > \frac{15}{40}$.
$\therefore \frac{4}{10}$ is larger.

Example 4

Skillsheet
Simplifying fractions
MAT08NASS10027

Simplify each fraction.

a $\frac{10}{15}$ **b** $\frac{36}{60}$

Solution

a $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$

Or on a calculator, enter: 10 $a^{b}/_{c}$ 15 =.

b $\frac{36}{60} = \frac{36 \div 6}{60 \div 6} = \frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$

or $\frac{36}{60} = \frac{36 \div 12}{60 \div 12} = \frac{3}{5}$

We can simplify in one step if we divide by 12, the HCF of 36 and 60.

Or on a calculator, enter: 36 $a^{b}/_{c}$ 60 =.

Exercise 6-01 Fractions

See Example 1

1 Convert each improper fraction into a mixed numeral or whole number.

a $\frac{3}{2}$ **b** $\frac{11}{3}$ **c** $\frac{9}{4}$

d $\frac{12}{5}$ **e** $\frac{20}{4}$ **f** $\frac{47}{11}$

See Example 2

2 Convert each mixed numeral into an improper fraction.

a $3\frac{1}{2}$ **b** $4\frac{1}{3}$ **c** $5\frac{3}{4}$ **d** $7\frac{2}{3}$

e $3\frac{4}{5}$ **f** $7\frac{2}{9}$ **g** $10\frac{1}{7}$ **h** $9\frac{3}{10}$

3 Write an improper fraction that can be converted into a whole number.

4 Which is larger: a proper fraction or a mixed numeral?

See Example 3

Worked solutions
Exercise 6-01
MAT08NAWS10045

5 For each pair of fractions, find the larger fraction.

a $\frac{1}{2}, \frac{1}{3}$ **b** $\frac{2}{3}, \frac{3}{4}$ **c** $\frac{3}{8}, \frac{1}{3}$

d $2\frac{3}{5}, 2\frac{7}{10}$ **e** $\frac{11}{15}, \frac{3}{5}$ **f** $\frac{5}{12}, \frac{2}{5}$

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6 Simplify each fraction.

See Example 4

a $\frac{5}{10}$ **b** $\frac{4}{12}$ **c** $\frac{12}{26}$ **d** $\frac{18}{24}$

e $\frac{15}{25}$ **f** $\frac{16}{28}$ **g** $\frac{32}{48}$ **h** $\frac{60}{100}$

i $\frac{44}{77}$ **j** $\frac{20}{35}$ **k** $\frac{21}{35}$ **l** $\frac{72}{80}$

7 Which one of these fractions is **not** equivalent to $\frac{10}{25}$? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{1}{5}$ **B** $\frac{2}{5}$ **C** $\frac{20}{50}$ **D** $\frac{40}{100}$

8 Convert each improper fraction into a simplified mixed numeral.

a $\frac{9}{6}$ **b** $\frac{18}{6}$ **c** $\frac{45}{10}$ **d** $\frac{36}{20}$

Worked solutions
Exercise 6-01
MAT08NAWS10045

6-02 Adding and subtracting fractions

Puzzle sheet
Adding and subtracting fractions
MAT08NAPS00018

Worksheet
Fraction arithmagons
MAT08NAWK00035

Summary

- To **add or subtract fractions**, convert them (if needed) so that they have the same denominator, then simply add or subtract just the numerators.
- To **add or subtract mixed numerals**, add or subtract the whole numbers and fractions separately.

Example 5

Evaluate each expression.

a $\frac{1}{3}+\frac{5}{6}$ **b** $\frac{5}{7}-\frac{2}{3}$ **c** $3-\frac{3}{4}$ **d** $1\frac{2}{3}+4\frac{1}{5}$

Solution

a Common denominator $= 3 \times 6 = 18$

$\frac{1}{3}=\frac{1\times 6}{3\times 6}=\frac{6}{18}$

$\frac{5}{6}=\frac{5\times 3}{6\times 3}=\frac{15}{18}$

$\frac{1}{3}+\frac{5}{6}=\frac{6}{18}+\frac{15}{18}$

$=\frac{21}{18}$

$=\frac{7}{6}$

$=1\frac{1}{6}$

Writing equivalent fractions using the common denominator.

Note that we multiply the numerator and denominator of each fraction by the denominator of the ***other*** **fraction**

OR Lowest common denominator of 3 and 6 = 6:

$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$

Writing equivalent fractions using the lowest common denominator.

$\frac{5}{6} = \frac{5}{6}$

$\frac{1}{3} + \frac{5}{6} = \frac{2}{6} + \frac{5}{6}$

$= \frac{7}{6}$

$= 1\frac{1}{6}$

OR on a calculator, enter: 1 $a^{b}/_{c}$ 3 + 5 $a^{b}/_{c}$ 6 =.

b Common denominator = $7 \times 3 = 21$:

$\frac{5}{7} = \frac{5 \times 3}{7 \times 3} = \frac{15}{21}$

Using a common denominator.

$\frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$

$\frac{5}{7} - \frac{2}{3} = \frac{15}{21} - \frac{14}{21}$

$= \frac{1}{21}$

OR on a calculator, enter: 5 $a^{b}/_{c}$ 7 − 2 $a^{b}/_{c}$ 3 =.

c $3 - \frac{3}{4} = 2 + 1 - \frac{3}{4}$

$= 2 + \frac{4}{4} - \frac{3}{4}$

$= 2\frac{1}{4}$

OR on a calculator, enter: 3 − 3 $a^{b}/_{c}$ 4 =.

d $4\frac{1}{5} - 1\frac{2}{3} = 4 - 1 + \frac{1}{5} - \frac{2}{3}$

$= 3 + \frac{3}{15} - \frac{10}{15}$

$= 3 - \frac{7}{15}$

$= 2 + \frac{15}{15} - \frac{7}{15}$

$= 2\frac{8}{15}$

OR on a calculator, enter: 4 $a^{b}/_{c}$ 1 $a^{b}/_{c}$ 5 − 1 $a^{b}/_{c}$ 2 $a^{b}/_{c}$ 3 =.

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Exercise 6-02 Adding and subtracting fractions

See Example 5

Worked solutions

Exercise 6-02

MAT08NAWS10046

1 Evaluate each expression.

a $\frac{1}{5}+\frac{3}{5}$ **b** $\frac{3}{4}-\frac{1}{4}$ **c** $\frac{3}{8}+\frac{2}{8}$ **d** $\frac{7}{8}+\frac{4}{8}$

2 Evaluate each expression.

a $\frac{2}{5}+\frac{3}{10}$ **b** $\frac{2}{3}-\frac{3}{7}$ **c** $\frac{3}{5}+\frac{1}{4}$ **d** $\frac{1}{2}-\frac{1}{4}$

e $\frac{1}{6}+\frac{3}{8}$ **f** $\frac{7}{10}-\frac{2}{3}$ **g** $\frac{4}{9}+\frac{5}{6}$ **h** $\frac{7}{8}-\frac{5}{12}$

i $\frac{5}{8}+\frac{1}{2}$ **j** $1-\frac{6}{7}$ **k** $4-\frac{4}{9}$ **l** $3-\frac{2}{5}$

3 Joe bought a length of timber to build fences for his garden beds. He used $\frac{1}{5}$ of the wood for the first garden and $\frac{3}{8}$ of the wood for the second garden. What fraction of the wood remains for the third garden?

4 What fraction goes in the blank? $\frac{7}{8}-$ ____ $=\frac{1}{4}$. Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{6}{4}$ **B** $\frac{8}{12}$ **C** $\frac{3}{8}$ **D** $\frac{5}{8}$

5 Copy and complete each blank with the correct fraction.

a $\frac{1}{4}+$ ___ $=\frac{2}{3}$ **b** ___ $-\frac{2}{5}=\frac{1}{6}$

6 A bottle is three-quarters full. One-third of the liquid is then poured out. What fraction remains in the bottle?

7 Danielle used half a sheet of adhesive plastic to cover her books, while Christina used $\frac{2}{5}$ of the same sheet. What fraction of the original sheet remains? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{3}{5}$ **B** $\frac{4}{5}$ **C** $\frac{1}{10}$ **D** $\frac{9}{10}$

8 What is the value of $2\frac{7}{8}+1\frac{3}{8}$? Select the correct answer **A**, **B**, **C** or **D**.

A $4\frac{1}{4}$ **B** $3\frac{4}{5}$ **C** $3\frac{1}{2}$ **D** $13\frac{1}{4}$

9 Evaluate each expression.

a $1\frac{1}{2}+\frac{2}{5}$ **b** $2\frac{1}{4}+1\frac{3}{8}$ **c** $7\frac{3}{8}+9\frac{1}{3}$ **d** $3\frac{4}{5}+1\frac{2}{9}$

e $6\frac{2}{5}-4\frac{7}{8}$ **f** $4\frac{3}{4}-2\frac{4}{5}$ **g** $2\frac{1}{3}-1\frac{1}{2}$ **h** $4\frac{3}{5}-2\frac{9}{10}$

10 In a magic square, each row, column and diagonal must add to the same total. Complete each magic square with appropriate fractions.

a

4	$1\frac{1}{2}$	
$\frac{1}{2}$		$4\frac{1}{2}$
3		

b

$\frac{1}{6}$		
	$\frac{2}{3}$	
	$\frac{1}{3}$	$1\frac{1}{6}$

Homework sheet
Fractions
MAT08NAHS10008

Worksheet
Multiplying and dividing fractions number grids
MAT08NAWK00036

Worksheet
Fractions and decimals review
MAT08NAWK00037

6-03 Multiplying and dividing fractions

Summary

- To **multiply fractions**, multiply the numerators and denominators separately, then simplify if possible (sometimes, it is easier to simplify the fractions first).
- To **divide by a fraction**, $\frac{a}{b}$, multiply by its reciprocal $\frac{b}{a}$.
- To **multiply or divide mixed numerals**, convert them to improper fractions first.

Example 6

Evaluate each expression.

a $\frac{3}{5} \times \frac{2}{7}$ **b** $\frac{2}{3} \times \frac{3}{8}$ **c** $\frac{4}{5} \div \frac{2}{3}$ **d** $1\frac{1}{2} \times 2\frac{2}{3}$

Solution

a $\frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7}$

$= \frac{6}{35}$

OR on a calculator, enter: 3 $a^b/_c$ 5 × 2 $a^b/_c$ 7 =

b $\frac{2}{3} \times \frac{3}{8} = \frac{2 \times 3}{3 \times 8}$

$= \frac{6}{24}$

$= \frac{1}{4}$

OR by simplifying first before multiplying:

$\frac{2}{3} \times \frac{3}{8} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \times \underset{4}{\cancel{8}}}$ Because $\frac{2}{8} = \frac{1}{4}$, $\frac{3}{3} = \frac{1}{1}$

$= \frac{1}{4}$

OR on a calculator, enter: 2 $a^b/_c$ 3 × 3 $a^b/_c$ 8 =

c $\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2}$

$= \frac{12}{10}$

$= \frac{6}{5}$

$= 1\frac{1}{5}$

OR by simplifying first before multiplying:

$\frac{4}{5} \div \frac{2}{3} = \frac{\overset{2}{\cancel{4}}}{5} \times \frac{3}{\underset{1}{\cancel{2}}} = \frac{6}{5} = 1\frac{1}{5}$

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OR on a calculator, enter: 4 $a^b/_c$ 5 ÷ 2 $a^b/_c$ 3 =

d $1\frac{1}{2} \times 2\frac{2}{3} = \frac{3}{2} \times \frac{8}{3}$

$= \frac{\cancel{3}^{1}}{\cancel{2}_{1}} \times \frac{\cancel{8}^{4}}{\cancel{3}_{1}}$

$= \frac{4}{1}$

$= 4$

OR on a calculator, enter: 1 $a^b/_c$ 1 $a^b/_c$ 2 × 2 $a^b/_c$ 2 $a^b/_c$ 3 =

Exercise 6-03 Multiplying and dividing fractions

1 Evaluate each product. *See Example 6*

a $\frac{1}{2} \times \frac{2}{3}$ **b** $\frac{2}{3} \times \frac{5}{7}$ **c** $\left(\frac{1}{8}\right)^2$ **d** $\frac{9}{10} \times \frac{5}{6}$

e $\frac{3}{5} \times \frac{3}{4}$ **f** $1\frac{1}{2} \times 3$ **g** $3\frac{1}{4} \times 3\frac{4}{5}$ **h** $1\frac{1}{3} \times 2\frac{1}{3}$

2 What is the value of $\frac{7}{10} \div \frac{1}{5}$? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{7}{50}$ **B** $1\frac{1}{5}$ **C** $1\frac{2}{5}$ **D** $3\frac{1}{2}$

Worked solutions
Exercise 6-03
MAT08NAWS10047

3 **a** What number's reciprocal is itself?

b What number does not have a reciprocal?

c What is the product of a number and its reciprocal?

d What is the reciprocal of the reciprocal of $\frac{1}{3}$?

4 Evaluate each quotient.

a $\frac{1}{7} \div \frac{2}{3}$ **b** $\frac{2}{5} \div \frac{3}{4}$ **c** $\frac{5}{9} \div \frac{5}{6}$ **d** $\frac{9}{10} \div \frac{3}{5}$

e $4 \div \frac{1}{4}$ **f** $6 \div \frac{2}{3}$ **f** $\frac{5}{8} \div 5$ **g** $\frac{2}{5} \div 6$

i $\frac{7}{8} \div \frac{1}{4}$ **j** $10 \div 3\frac{1}{7}$ **k** $3\frac{1}{2} \div 1\frac{1}{4}$ **l** $3\frac{3}{4} \div 1\frac{1}{10}$

5 From a box of chocolates, Lindy takes $\frac{3}{8}$ of the chocolates and shares them equally among her four children. What fraction of the box of lollies does each child receive?

6 Copy and fill in each blank.

a $\frac{3}{4} \times __ = \frac{21}{32}$ **b** $__ \times \frac{1}{5} = \frac{2}{15}$

c $__ \div \frac{2}{3} = \frac{1}{7}$ **d** $\frac{4}{15} \div __ = \frac{8}{21}$

7 **a** When a number is divided by a proper fraction, does the number increase or decrease? Give reasons for your answer.

b When a number is multiplied by an improper fraction, does the number increase or decrease? Give reasons for your answer.

Worksheet
Fractions wall
MAT08NAWK10050

Worksheet
Decimals wall
MAT08NAWK10051

Worksheet
Percentages wall
MAT08NAWK10052

Worksheet
Fractions, decimals and percentages
MAT08NAWK10053

Skillsheet
Multiplying and dividing by 10, 100, 1000
MAT08NASS10028

6-04 Percentages, fractions and decimals

A **percentage** is a fraction whose denominator is 100.

Example 7

Convert each percentage into a simplified fraction.

a 55% **b** 130% **c** $37\frac{1}{2}\%$

Solution

a $55\% = \frac{55}{100}$
$= \frac{11}{20}$

b $130\% = \frac{130}{100}$
$= \frac{13}{10}$
$= 1\frac{3}{10}$

c $37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100}$
$= \frac{37\frac{1}{2} \times 2}{100 \times 2}$
$= \frac{75}{200}$
$= \frac{3}{8}$

Puzzle sheet
Percentages to decimals
MAT08NAPS00015

Puzzle sheet
Percentominoes
MAT08NAPS00016

Example 8

Convert each percentage into a decimal.

a 8% **b** 43.6% **c** $18\frac{1}{2}\%$

Solution

a $8\% = \frac{8}{100}$
$= 8 \div 100$
$= 0.08$

b $43.6\% = \frac{43.6}{100}$
$= 43.6 \div 100$
$= 0.436$

To convert a percentage into a decimal mentally, move the decimal point 2 places to the left

c $18\frac{1}{2}\% = \frac{18\frac{1}{2}}{100}$
$= 18.5 \div 100$
$= 0.185$

Summary

To convert a fraction or a decimal into a percentage, multiply it by 100%

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Example 9

Convert each fraction into a percentage.

a $\frac{11}{20}$ **b** $\frac{7}{8}$

Solution

a $\frac{11}{20} = \frac{11}{20} \times 100\%$
$= 55\%$

b $\frac{7}{8} = \frac{7}{8} \times 100\%$
$= 87\frac{1}{2}\%$

Example 10

Convert each decimal into a percentage.

a 0.65 **b** 0.267

Solution

a $0.65 = 0.65 \times 100\%$
$= 65\%$

b $0.267 = 0.267 \times 100\%$
$= 26.7\%$

To convert a decimal into a percentage mentally just move the decimal point 2 places to the right

Summary

To order fractions, percentages and decimals, convert each into a percentage first.

Example 11

Arrange in ascending order: 0.667, 66%, $\frac{7}{11}$.

Solution

Convert all numbers into percentages.
$0.667 \times 100\% = 66.7\%$
$66\% = 66\%$

$\frac{7}{11} \times 100\% = 63\frac{7}{11}\%$

Arrange the percentages from smallest to largest: $63\frac{7}{11}\%$, 66% and 66.7%
So, in ascending order, the numbers are $\frac{7}{11}$, 66% and 0.667.

Exercise 6-04 Percentages, fractions and decimals

See Example 7

1 Convert each percentage into a simplified fraction.

a 60%	**b** 75%	**c** 31%	**d** 8%
e 30%	**f** 85%	**g** 99%	**h** 3%
i 160%	**j** 135%	**k** 25%	**l** 250%

See Example 8

2 Which decimal is equal to $62\frac{1}{2}\%$? Select the correct answer **A**, **B**, **C** or **D**.

A 62.12 **B** 62.5 **C** 0.625 **D** 0.0625

Worked solutions
Exercise 6-04
MAT08NAWS10048

3 Convert each percentage to a decimal.

a 18%	**b** 82%	**c** 2%	**d** 50%
e 120%	**f** 51.1%	**g** 79%	**h** $12\frac{1}{2}\%$
i 16.3%	**j** 4%	**k** 18.7%	**l** $5\frac{1}{4}\%$

See Example 9

4 Convert each fraction into a percentage.

a $\frac{17}{100}$	**b** $\frac{7}{10}$	**c** $\frac{13}{50}$	**d** $\frac{11}{20}$
e $\frac{5}{8}$	**f** $\frac{24}{25}$	**g** $\frac{2}{3}$	**h** $\frac{5}{4}$
i $1\frac{2}{5}$	**j** $\frac{27}{40}$	**k** $\frac{1}{16}$	**l** $\frac{4}{9}$

See Example 10

5 Convert each decimal into a percentage.

a 0.38	**b** 0.55	**c** 0.96	**d** 0.625
e 0.08	**f** 0.054	**g** 0.6	**h** 0.003
i 1.9	**j** 0.405	**k** 1.26	**l** 0.114

6 Copy and complete this table.

	Fraction	Decimal	Percentage
a		0.65	
b		0.6	
c			20%
d			84%
e	$\frac{1}{2}$		
f	$\frac{1}{8}$		
g			36%
h	$\frac{5}{8}$		
i		0.73	
j	$\frac{1}{3}$		
k			$66\frac{2}{3}\%$

See Example 11

7 For each pair of numbers, determine which is larger.

a 75% and 0.73	**b** $\frac{4}{25}$ and 14%	**c** $\frac{2}{3}$ and 0.64
d 18% and $\frac{1}{7}$	**e** 0.22 and $\frac{1}{4}$	**f** 55% and $\frac{6}{11}$
g 0.93 and $\frac{19}{20}$	**h** 60% and $\frac{13}{25}$	**i** $\frac{1}{6}$ and 0.08

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Worked solutions
Exercise 6-04
MAT08NAWS10048

8 Arrange each set of numbers in ascending order.

a $\frac{4}{5}$, 78%, 0.75, $\frac{9}{11}$ **b** 22%, $\frac{1}{4}$, 0.29, $\frac{7}{20}$ **c** 0.62, $\frac{3}{5}$, 57%, 0.605

9 Which list of numbers is arranged in descending order? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{4}{11}$, $\frac{2}{5}$, 0.41, 43% **B** $\frac{2}{5}$, 0.41, $\frac{4}{11}$, 43%

C 43%, 0.41, $\frac{2}{5}$, $\frac{4}{11}$ **D** 0.41, $\frac{4}{11}$, 43%, $\frac{2}{5}$

10 Arrange each set of numbers in descending order.

a $\frac{9}{20}$, 0.47, $\frac{2}{5}$, 43% **b** 0.08, 86%, $\frac{21}{25}$, 0.88 **c** 0.905, $\frac{19}{20}$, 91%, $\frac{9}{10}$

Mental skills 6A Maths without calculators

Finding 10%, 20% and 5%

To find 10% or $\frac{1}{10}$ of a number, simply divide the number by 10 by moving the decimal point one place to the left.

1 Study each example.

a $10\% \times 150 = 15\overset{\curvearrowleft}{0}. = 15$

b $10\% \times \$1256.80 = \$125\overset{\curvearrowleft}{6}.8 = \$125.68$

c $10\% \times 4917 = 491\overset{\curvearrowleft}{7}. = 491.7$

d $10\% \times \$48.55 = \$4\overset{\curvearrowleft}{8}.55 = \$4.885$

2 Now find 10% of each amount.

a 190	**b** \$75	**c** 875	**d** \$202
e \$37.60	**f** 400	**g** \$9.25	**h** 896
i \$2700	**j** \$3.80	**k** \$1527.60	**l** \$72.50
m 3154	**n** \$10.70	**o** 426	**p** \$24 317.60

20% is 10% doubled so to find 20% of a number, first find 10% then double it.

3 Study each example.

a $20\% \times 700$
$10\% \times 700 = 70$
$\therefore 20\% \times 700 = 70 \times 2 = 140$

b $20\% \times \$876$
$10\% \times \$876 = \$87.60$
$\therefore 20\% \times \$876 = \$87.60 \times 2 = \$175.20$

c $20\% \times 325$
$10\% \times 325 = 32.5$
$\therefore 20\% \times 325 = 32.5 \times 2 = 65$

d $20\% \times \$38.50$
$10\% \times \$38.50 = \$3.85$
$\therefore 20\% \times \$38.50 = \$3.85 \times 2 = \$7.70$

4 Now find 20% of each amount.

a 50	**b** 620	**c** \$2450	**d** \$8.60
e 72	**f** \$12 700	**g** 390	**h** \$5.80
i \$45	**j** \$84	**k** \$4600	**l** 320

5% is half of 10%, so to find 5% of a number first find 10% then divide it by 2.

5 Study each example.

a $5\% \times 180$
$10\% \times 180 = 18$
$\therefore 5\% \times 180 = 18 \div 2 = 9$

b $5\% \times \$76$
$10\% \times \$76 = \$7.60$
$\therefore 5\% \times \$76 = \$7.60 \div 2 = \$3.80$

c $5\% \times 120$
$10\% \times 120 = 12$
$\therefore 5\% \times 12 = 12 \div 2 = 6$

d $5\% \times \$142.20$
$10\% \times \$142.20 = \$142.22$
$\therefore 5\% \times \$142.20 = \$14.22 \div 2 = \$7.11$

6 Now find 5% of each amount.

a 2000 **b** \$12 **c** 50 **d** \$27
e \$36.80 **f** \$84 **g** 800 **h** 130
i \$9.60 **j** \$138 **k** \$72 **l** 840

Puzzle sheet
Percentage crossnumber
MAT08NAPS00019

Puzzle sheet
Percentages
MAT08NAPS00039

6-05 Fraction and percentage of a quantity

Example 12

Find:

a $\frac{1}{5} \times 45$ **b** $\frac{7}{8} \times \$32$ **c** $\frac{1}{3}$ of one year **d** $\frac{4}{5}$ of 2 kg (in grams)

Solution

a $\frac{1}{5} \times 45 = 45 \div 5$
$= 9$

b $\frac{7}{8} \times \$32 = \left(\frac{1}{8} \times \$32\right) \times 7$
$= \$4 \times 7$
$= \$28$

c $\frac{1}{3}$ of 1 year $= \frac{1}{3} \times 12$ months
$= 4$ months

Convert 1 year to 12 months.

d $\frac{4}{5}$ of 2 kg $= \frac{4}{5} \times 2000$ g
$= \left(\frac{1}{5} \times 2000 \text{ g}\right) \times 4$
$= 400 \text{ g} \times 4$
$= 1600$ g

Convert 2 kg to 2000 g.

Summary

To find a **percentage of a quantity**, calculate:

$\frac{\text{percentage}}{100} \times \text{quantity}$ or percentage $\div$ 100 $\times$ quantity

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Example 13

Find:

a 8% of \$400 **b** 12.5% of 1 hour (in minutes) **c** 20% of 3 m (in centimetres).

Solution

a $8\% \text{ of } \$400 = \frac{8}{100} \times \$400$ or $8 \div 100 \times \$400$ or $0.08 \times \$400$

$= \$32$

b $12.5\% \text{ of } 1 \text{ h} = 12.5\% \times 60 \text{ min}$ Convert 1 h to 60 min

$= \frac{12.5}{100} \times 60 \text{ min}$ or $12.5 \div 100 \times 60$ or 0.125×60

$= 7.5 \text{ min}$

c $20\% \text{ of } 3 \text{ m} = 20\% \times 300 \text{ cm}$ Convert 3 m to 300 cm

$= \frac{20}{100} \times 300 \text{ cm}$ or $20 \div 100 \times 300$ or 0.2×300

$= 60 \text{ cm}$

Exercise 6-05 Fraction or percentage of a quantity

1 Find: *See* Example 12

a $\frac{3}{5} \times 40$ **b** $\frac{1}{4} \times 28$ **c** $\frac{1}{6} \times 24$

d $\frac{2}{3} \times 15$ **e** $\frac{7}{10} \times 60$ **f** $\frac{5}{8} \times 16$

g $\frac{3}{4}$ of 1 km (in metres) **h** $\frac{1}{3}$ of 1 day (in hours) **i** $\frac{2}{5}$ of 1 L (in mL)

j $\frac{1}{8}$ of 1 t (in kgs) **k** $\frac{5}{6}$ of 1 year (in months) **l** $\frac{7}{12}$ of 1 hour (in minutes)

2 What is 35% of \$75? Select the correct answer **A**, **B**, **C** or **D**.

A \$40 **B** \$110 **C** \$26.25 **D** \$48.75

3 Find: *See* Example 13

a 11% of \$500 **b** 2% of 250 kg **c** 13% of 150 L

d 21% of 600 cm **e** 15% of \$450 **f** 5% of 5700 g

g 42% of 1128 m **h** 112% of 256 km **i** 19.4% of 785 mL

j 7.1% of \$220 **k** 23.6% of \$380 **l** 11.3% of 403 kg

m 110% of 95 **n** 150% of 302 **o** 130% of \$2010

p 105% of 120 m **q** 170% of 350 g **r** 115% of \$400

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4 Taffy the cat had a mass of 2.7 kg when she was found. If her mass increased by $\frac{1}{4}$, how much did she gain?

5 A **discount** is a saving made between the original price of an item and the cheaper price.

a How much do you *save* if you get a 25% discount on a $420 games system?

b What would be the discount price of the games system?

Worked solutions
Exercise 6-05
MAT08NAWS10049

6 An Airbus can seat 351 passengers. If the plane flew with $\frac{2}{3}$ of the seats occupied, how many passengers were on the flight?

7 Calculate each of the following. (Convert to a smaller unit first.)

a 12% of 8 m	**b** 35% of 1 tonne	**c** 20% of 12 hours
d $2\frac{1}{2}$% of 3 litres	**e** 72% of 150 kg	**f** $7\frac{1}{2}$% of 12 km
g 35% of 10 days	**h** $12\frac{1}{2}$% of 4 hours	**i** 67.5% of 40 g

8 12% of the 525 swimmers at the swimming carnival wore black caps. How many swimmers wore black caps?

9 Haroula earns $17.20 an hour in her part-time job. If she is given a 5% pay increase, by how much does her hourly rate increase?

10 If 42% of the 650 road deaths occurred on country roads, how many road deaths happened in the country?

11 In a city of 3 million people, 1% of the population are doctors. How many doctors are there? Select **A**, **B**, **C** or **D**.

A 3 **B** 3000 **C** 300 **D** 30 000

12 One-twentieth of the cars produced in a factory were found to have steering defects. If 340 cars were produced, how many had steering defects?

13 How many children are there in a crowd of 40 530 if $\frac{3}{5}$ are children?

14 If 70% of the seats at a rock concert must be sold to make a profit, how many must be sold in a 2000-seat theatre?

15 Copy and complete.

a 20% × _____ = $18 **b** 15% × _____ = $75

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Technology Discounts

In this activity, you will calculate the discount and sale price of items in a department store given their original price.

1 Enter the following data into a spreadsheet. Make the headings in row 1 **bold**.

	A	B	C	D	E
1	Item	Original Price	% Discount	Discount	Sale Price
2	Backpack	$39.95	10%	=B2*C2	
3	DVD	$15.00	5%		
4	Beach towel	$20	30%		
5	Book	$29.95	15%		
6	Swimwear	$60	20%		
7	MP3 player	$145	10%		

2 Cell D2 shows the formula for calculating the discount on the backpack. Right-click on D2 and **Fill Down** to copy the formula down to cell D7. This will calculate the discount that applies to each item. To add $ signs to the values, with D2 to D7 still selected, right-click and select **Format Cells** and **Currency** with **two decimal places**.

3 In cell E2, enter the formula =**B2-D2** to calculate the sale price of the backpack. Use **Fill Down** to calculate the sale prices of the remaining items. To add $ signs to the values, with E2 to E7 still selected, right-click and select **Format Cells** and **Currency** with **two decimal places**.

6-06 Expressing amounts as fractions and percentages

Worksheet
Percentages
MAT08NAWK00061

Technology worksheet
Online discounts
MAT08NACT10013

Technology worksheet
Coffee shop sales
MAT08NACT10003

Quiz
Fractions and decimals
MAT08NAQZ00005

Summary

To write an amount as a fraction of a whole amount:

- write the amount in the numerator of the fraction
- write the whole amount (total) in the denominator

$$\text{Fraction} = \frac{\text{amount}}{\text{whole amount}}$$

To write an amount as a percentage of a whole amount:

- write the amount in the numerator of a fraction
- write the whole amount (total) in the denominator
- multiply by 100%

$$\text{Percentage} = \frac{\text{amount}}{\text{whole amount}} \times 100\%$$

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Example 14

There were 250 people at the school fete, and 160 of them were children.

a What fraction of the people at the fete were children?

b What percentage of the people at the fete were children?

Solution

a $\text{Fraction} = \dfrac{\text{Number of children}}{\text{Total number of people}}$

$= \dfrac{160}{250}$

$= \dfrac{16}{25}$

b $\text{Percentage} = \dfrac{160}{250} \times 100\%$

$= 64\%$

Example 15

Express 36 minutes as:

a a fraction of an hour

b a percentage of an hour

Solution

Quantities need to be expressed in the same units, so change 1 hour to 60 minutes.

a $\text{Fraction} = \dfrac{36}{60} = \dfrac{3}{5}$

b $\text{Percentage} = \dfrac{36}{60} \times 100\%$

$= 60\%$

Exercise 6-06 Expressing amounts as fractions and percentages

See Example 14

1 Convert each test mark into a simplified fraction.

a 50 out of 100 **b** 38 out of 50 **c** 87 out of 100

d 8 out of 12 **e** 5 out of 20 **f** 45 out of 120

2 Convert each test mark in question **1** into a percentage.

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Worked solutions
Exercise 6-06
MAT08NAWS10050

3 A hockey team scored 8 goals. If one player scored 5 of them, what is this as a percentage of the team score?

4 In a class of 25 students, 6 ride bikes, 10 walk to school and the rest catch the bus.
- **a** What fraction walk to school?
- **b** What percentage catch the bus?

5 Wakeel answered 21 questions correctly out of 24 in his driving test. If the pass mark is 95%, did he pass?

6 The World Cup cricket final attracted 86 000 people to the game, but 94 500 tickets were sold before the match.
- **a** What fraction of the sold tickets were used?
- **b** What percentage (correct to the nearest whole number) of the sold tickets were not used?

7 Tania earns $1340 a week. She pays $428.80 in tax and saves $180 a week.
- **a** What percentage of Tania's earnings is paid in tax?
- **b** What fraction of her earnings does Tania save?
- **c** What percentage (correct to one decimal place) of her earnings does Tania save?

8 Louise sells a house for $458 000. If the real estate agent is paid a commission of $22 900, what percentage of the sale price does the agent receive?

9 The Great Gals are having a sale on microwave ovens. The sale price of each oven is listed below, along with the discount.

i

$149
Save $20

ii

$179
Save $26

iii

$219
Save $30

- **a** Calculate the original price of each microwave oven.
- **b** Calculate, to one decimal place, the percentage discount on each oven.
- **c** Which oven has the greatest percentage discount?

10 Joshua bought a pair of roller skates for $180 and sold them at a profit of $45. What is the profit as a percentage? Select the correct answer **A**, **B**, **C** or **D**.

A 25% **B** 4% **C** 45% **D** 20%

11 Express each measurement as a simplified fraction.

See **Example 15**

a 5 min of 1 h	**b** 250 mL of 1 L	**c** 700 m of 1 km	**d** 230 kg of 1 t
e 75c of $6	**f** 40 min of 4h	**g** 300 g of 2 kg	**h** $3.80 of $14
i 12 h of 4 days	**j** 75 mm of 20 cm	**k** 800 m of 1.5 km	**l** 400 mL of 3.5 L

12 Express each measurement in question **11** as a percentage.

13 What is 40 minutes as a percentage of 2 hours? Select the correct answer **A**, **B**, **C** or **D**.

A 20% **B** 5% **C** $33\frac{1}{3}$% **D** 0.33%

14 A football team is scoring well in its matches if its points percentage is over 100, according to the formula:

$$\text{Points percentage} = \frac{\text{Points for}}{\text{Points against}} \times 100$$

where 'points for' are the total number of points the team has scored and 'points against' are the total number of points the other teams have scored when playing against that team.

a Calculate, correct to two decimal places, the points percentage for each team listed in the table.

Team	Points for	Points against	Team	Points for	Points against
Broncos	391	313	Raiders	405	466
Bulldogs	410	366	Roosters	296	479
Cowboys	383	452	Sea Eagles	434	235
Dragons	321	395	Sharks	328	279
Eels	407	352	Storm	478	207
Knights	332	382	Tigers	498	642
Panthers	357	469	Titans	319	399
Rabbitohs	275	283	Warriors	688	454

b What are the top four teams based on these results?

Just for the record — Dollars

Australia, the USA, Hong Kong and parts of Central America use the dollar as their unit of currency. The word 'dollar' actually originated in the town of Jachymov in Czechoslovakia, where the coins minted from its silver mine were called 'Joachimsthaler'. Coins in various German states from the 16th to the 19th centuries were called 'taler', which later became 'daler'. The early Spanish settlers introduced the 'Philippe dolers' into North America, and this later became the word 'dollar' that we know today.

Find out which countries use dollars as their unit of currency.

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Investigation: Percentages in sport

Percentages are a good way of comparing sporting performances. Commentators often give statistics in the form of percentages so that we can compare the success of teams or players.

1 In netball, goal shooters rarely have the same number of shooting chances. To work out their success rates, we must use percentages to compare the number of shooting chances with the number of successful shots.
The following statistics were recorded in a match between Australia and New Zealand.

	Australia		New Zealand	
	Shooting chances	**Goals**	**Shooting chances**	**Goals**
1st quarter	15	11	9	9
2nd quarter	25	17	14	13
3rd quarter	18	14	23	20
4th quarter	27	20	18	17

a Calculate as a percentage to one decimal place:
i each team's success rate for each quarter
ii the total success rate for each team.

b Comment on the relationship between the success rates and the final scores. What can you say about each team?

2 At any stage during the season in the National Basketball League (NBL), the teams will have played different numbers of games. The highest position on the ladder is awarded to the team with the highest **wins percentage,** which is calculated as follows:

$$\text{Wins percentage} = \frac{\text{Number of games won}}{\text{Number of games played}} \times 100\%$$

At one stage of a season the teams had won and lost the number of games shown below.

Team	Won	Lost
Adelaide	6	6
Cairns	8	7
Gold Coast	7	4
Melbourne	6	9
New Zealand	9	4
Perth	3	11
Sydney	11	3
Townsville	8	4
Wollongong	2	12

Calculate the wins percentage for each team and construct the 'NBL Ladder' showing the teams in the correct order.

Worksheet
Percentage shortcuts
MAT08NAWK10054

Worksheet
Discounts and special offers
MAT08NAWK10055

Video tutorial
Percentage increase and decrease
MAT08NAVT10010

Homework sheet
Percentages 1
MAT08NAHS10009

Puzzle sheet
Discounts
MAT08NAPS00040

Worksheet
Discounts
MAT08NAWK00062

6-07 Percentage increase and decrease

- **Percentage increase** means to increase (make bigger) a quantity by a percentage
- **Percentage decrease** means to decrease (make smaller) a quantity by a percentage

Example 16

Increase \$200 by 7%.

Solution

Method 1

$7\%\ \text{of}\ \$200 = \frac{7}{100} \times \$200$ or $7 \div 100 \times \$200$ or $0.07 \times \$200$

$= \$14$

$\$200 + \$14 = \$214$

Method 2

$(100\% + 7\%)\ \text{of}\ \$200 = 107\%\ \text{of}\ \$200$ Increasing by 7% is the same as calculating 107%

$= \frac{107}{100} \times \$200$ or $107 \div 100 \times \$200$ or $1.07 \times \$200$

$= \$214$

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Example 17

Decrease $150 by 12%.

Solution

Method 1

$12\% \text{ of } \$150 = \frac{12}{100} \times \$150$ or $12 \div 100 \times \$150$ or $0.12 \times \$150$

$= \$18$

$\$150 - \$18 = \$132$

Method 2

$(100\% - 12\%) \text{ of } \$150 = 88\% \text{ of } \$150$ Decreasing by 12% is the same as calculating 88%

$= \frac{88}{100} \times \$150$ or $88 \div 100 \times \$150$ or $0.88 \times \$150$

$= \$132$

Example 18

The price of a watch increases by 15%. If its original price was $35, find its new price.

Solution

Method 1

$15\% \text{ of } \$35 = \$5.25$

$\$35 + \$5.25 = \$40.25$

Method 2

$(100\% + 15\%) \text{ of } \$35 = 115\% \text{ of } \$35$

$= \$40.25$

Example 19

Find the price of a computer game system, originally priced at $420, after a 9% discount.

Solution

Method 1

$9\% \text{ of } \$420 = \$37.80$

$\$420 - \$37.80 = \$382.20$

Method 2

$(100\% - 9\%) \text{ of } \$420 = 91\% \text{ of } \$420$

$= \$382.20$

Exercise 6-07 Percentage increase and decrease

See Example 16

1 Increase:

a \$150 by 5% **b** 400 by 20% **c** 60 km by 22%
d \$2500 by 6% **e** 95 kg by 60% **f** 10 L by 33%

See Example 17

2 Decrease:

a \$440 by 60% **b** 120 by 15% **c** 110 kg by 8%
d \$325 by 25% **e** 2000 L by 38% **f** \$1570 by 3%

See Example 18

3 Wooden chairs cost \$172 to make. Calculate the selling price if the chairs are marked up by 35% when sold.

4 Julie buys a bike for \$2700 and sells it a year later, making a 15% profit.

a How much profit did Julie make?

b What was the selling price of the bike?

5 Geeva's weekly pay of \$980 increased by 4.5%. What is his new pay?

6 Yumi's height of 168 cm increased by 2%. What is her new height?

7 A department store has a mark-up of 200% on clothing. If the store buys a vest for \$12, what will be its selling price after the mark-up? Select the correct answer **A**, **B**, **C** or **D**.

A \$32 **B** \$14.40 **C** \$24 **D** \$36

See Example 19

8 A car dealer offers a 15% discount on all new car purchases. What would you pay for a car marked at \$21 990?

9 Calculate the sale price of a pair of sports shoes that cost \$165 after a discount of 12%.

10 After speed cameras were installed, the road toll of 840 deaths decreased by 5%. What is the new road toll?

11 Aaron bought a house for \$464 000 and sold it 10 years later, making a 150% profit. Calculate the selling price of the house.

12 What is the new price of a pair of jeans worth \$75 if they are discounted by 25%? Select the correct answer **A**, **B**, **C** or **D**.

A \$50 **B** \$56.25 **C** \$18.75 **D** \$55

13 A netball sells for \$45, but you receive 11% discount if you buy two. What is the discounted price of two netballs?

14 The owner of a store buys a DVD recorder for \$470. She adds a mark-up of \$70 but, at sale time, offers a 30% discount.

a Find the selling price after the mark-up.

b Find the discounted price at sale time.

c How much loss did the store owner incur on the DVD recorder?

d Calculate the percentage loss correct to one decimal place.

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15 Find the number which if increased by 18% gives 767.

16 Winter coats priced at $830 were reduced by 15% at the end-of-season sale. On the last weekend of the sale, they were reduced by a further 40% off the discounted price. What was the final price of a coat?

Worked solutions
Exercise 6-07
MAT08NAWS10051

17 Find the number which if decreased by 40% gives 306.

18 A digital tablet with an original price of $680 has 10% GST added to it. It is then sold at an end-of-year sale for '10% off'. Is the sale price of the tablet more than, less than, or equal to its original price? Justify your answer by calculation.

6-08 Percentages without calculators

Skillsheet
Mental percentages
MAT08NASS10029

Worksheet
Percentages without calculators
MAT08NAWK10057

If we know the equivalent fractions for common percentages, then we can find percentages mentally.

Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{1}{10}$	$\frac{1}{8}$
Percentage	25%	50%	75%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$	20%	40%	60%	10%	$12\frac{1}{2}\%$

Example 20

Calculate each percentage mentally.

a $33\frac{1}{3}\%$ of \$1800 **b** $12\frac{1}{2}\%$ of 72 kg

Solution

a $33\frac{1}{3}\%$ of $\$1800 = \frac{1}{3}$ of $\$1800$
$= \$600$

b $12\frac{1}{2}\%$ of 72 kg $= \frac{1}{8}$ of 72 kg
$= 9$ kg

Example 21

a Increase \$470 by 10%.
b Decrease \$75 by 20%.

Solution

a 10% of $\$470 = \frac{1}{10} \times \$470$
$= \$47$
$\$470 + \$47 = \$517$

b 20% of $\$75 = \frac{1}{5} \times \$75$ (or $10\% \times \$75 \times 2$)
$= \$15$
$\$75 - \$15 = \$60$

Note: More examples and exercises on increasing and decreasing an amount by a percentage mentally can be found in **Mental skills 6B: Percentage increase and decrease** following **Exercise 6-08**.

Exercise 6-08 Percentages without calculators

Complete this exercise without using a calculator.

See Example 20

1 Find 25% of each amount.

a 32 kg **b** 180 mm **c** 24 L **d** \$1000

2 Find each percentage.

a 50% of 36 m **b** 75% of 12 g **c** $12\frac{1}{2}\%$ of 24 h
d 20% of 5 L **e** $66\frac{2}{3}\%$ of \$30 **f** 60% of 20 km
g 25% of 20 mL **h** $33\frac{1}{3}\%$ of 18 days **i** 75% of 100 kg

Worked solutions
Exercise 6-08
MAT08NAWS10052

3 Find 10% of each amount.

a 60 tonnes **b** 40 hours **c** 300 sheep **d** \$90

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4 Find each percentage.

a 10% of 70 **b** 50% of \$80 **c** 40% of 120 minutes
d 30% of \$12 **e** 60% of 20 cm **f** 20% of 400 mL

5 What is 5% of 80 days? Select the correct answer **A**, **B**, **C** or **D**.

A 8 days **B** 4 days **C** 40 days **D** 80 days

6 Find $12\frac{1}{2}\%$ of each amount.

a \$72 **b** 40 kg **c** 160 kL

7 Increase:

a \$80 by 25% **b** 140 by 10% **c** \$150 by $33\frac{1}{3}\%$
d 28 by 50% **e** \$240 by $12\frac{1}{2}\%$ **f** 45 by 20%

See **Example 21**

8 Decrease:

a \$310 by 10% **b** 120 by 20% **c** \$80 by $12\frac{1}{2}\%$
d 60 by 5% **e** \$900 by $66\frac{2}{3}\%$ **f** 2000 by 75%

Worked solutions
Exercise 6-08
MAT08NAWS10052

Mental skills 6B Maths without calculators

Percentage increase and decrease

The fraction equivalents of commonly-used percentages can help us when we need to increase or decrease a number by a percentage.

Percentage	10%	$12\frac{1}{2}\%$	20%	25%	$33\frac{1}{3}\%$	50%	$66\frac{2}{3}\%$	75%
Fraction	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$

1 Consider each example.

a Increase 80 by 20%

$20\% \text{ of } 80 = \frac{1}{5} \times 80$
$= 80 \div 5$
$= 16$
$80 + 16 = 96$

or

$10\% \text{ of } 80 = 8$
$20\% \text{ of } 80 = 8 \times 2$
$= 16$

b Increase \$36 by $66\frac{2}{3}\%$

$66\frac{2}{3}\% \text{ of } 36 = \frac{2}{3} \times 36$
$= \$36 \div 3 \times 2$
$= \$12 \times 2$
$= \$24$
$\$36 + \$24 = \$60$

2 Now increase:

a \$280 by 10% **b** 45 by $33\frac{1}{3}\%$ **c** 25 by 20% **d** \$400 by $12\frac{1}{2}\%$
e 64 by 50% **f** \$72 by 25% **g** \$55 by 10% **h** 90 by $66\frac{2}{3}\%$
i 120 by 75% **j** \$80 by 5% **k** \$250 by 20% **l** 70 by 40%

3 Consider each example.

a Decrease 225 by $33\frac{1}{3}\%$

$$33\frac{1}{3}\% \text{ of } 225 = \frac{1}{3} \times 225$$
$$= 225 \div 3$$
$$= 75$$
$$225 - 75 = 150$$

b Decrease \$70 by 15%

$$10\% \text{ of } \$70 = \frac{1}{10} \times \$70 = \$7$$
$$\therefore 5\% \text{ of } \$70 = \frac{1}{2} \times \$7 = \$3.50$$
$$\therefore 15\% \text{ of } \$70 = (10\% \times \$70) + (5\% \times \$70)$$
$$= \$7 + \$3.50$$
$$= \$10.50$$
$$\$70 - \$10.50 = \$59.50$$

4 Now decrease:

a \$480 by 25% **b** 60 by $33\frac{1}{3}\%$ **c** 110 by 20% **d** \$25 by 10%
e 900 by 50% **f** \$72 by $12\frac{1}{2}\%$ **g** \$320 by 75% **h** 150 by $66\frac{2}{3}\%$
i \$63 by 20% **j** \$100 by $12\frac{1}{2}\%$ **k** 250 by 10% **l** \$150 by 30%

Worksheet
The unitary method
MAT08NAWK10056

Video tutorial
The unitary method
MAT08NAVT10011

6-09 The unitary method

If we know the percentage of an amount, but not the actual amount, we can use the **unitary method** to find the whole amount. The word **unit** means 'one,' and with the unitary method we find 1% first.

Example 22

If 15% of an amount is \$75, what is the amount?

Solution

15% of amount = \$75

1% of amount = \$75 ÷ 15 — Find 1% first by dividing by 15.
= \$5

100% of amount = \$5 × 100 — Then find the whole amount by multiplying by 100.
= \$500

The amount is \$500. — Check: 15% × \$500 = \$75

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Summary

When using the **unitary method** to find an amount given a **percentage** of the amount:

- first find 1% of the amount by dividing by the known percentage
- then multiply by 100 to find the whole amount (100%).

Example 23

Farmer Brown lost 30% of her sheep in a flood. If she had 560 sheep left, how many sheep did she have before the flood?

Solution

Because 30% of the total were lost, $100\% - 30\% = 70\%$ of the total were left.

So: 70% of the total $= 560$ sheep

1% of the total $= 560 \div 70$

$= 8$ sheep

100% of the total $= 8 \times 100$

$= 800$ sheep

Farmer Brown had 800 sheep before the flood. Check: $70\% \times 800 = 560$

Exercise 6-09 The unitary method

See Example 22

1 What is the whole amount if:

a 10% of it is \$70?	**b** 25% of it is \$140?	**c** 17% of it is \$782?
d 8.5% of it is \$161.50?	**e** 16% of it is 64 cm?	**f** 20% of it is 56 kg?
g 42% of it is 1.26 m?	**h** 30% of it is 45 minutes?	**i** 70% of it is \$280?
j 110% of it is \$396?	**k** 45% of it is 180 kg?	**l** 120% of it is 132 minutes?

2 The deposit of 15% on a kitchen stove is \$180. What is the price of the stove?

3 There were 51 870 men in the crowd at the football match. If this was 65% of the crowd, what was the total attendance?

Worked solutions
Exercise 6-09
MAT08NAWS10053

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4 Last year Joanna paid $23 009.28 in income tax. If this represents 28% of her income, calculate her income.

See Example 23

5 Joe sold his car for $12 200, making a loss of 20% on the original price. What was the original price?

Worked solutions
Exercise 6-09
MAT08NAWS10053

6 A cake decorator makes 18% profit on cakes she decorates. If she sells a cake for $330.40, what was the original cost of the cake?
Select the closest answer **A**, **B**, **C** or **D**.
A $184 **B** $238 **C** $271 **D** $280

7 A town's population increased by 5%, or 1200 people. What was the old population?

8 The interest earned on an investment is $168. If the interest rate is 5.6%, how much was invested?

9 Shania buys a dress that has been discounted by 25%. If the sale price is $225, what was the original price of the dress?

10 Matthew buys a game system for $348 at a discount of 20%. What was the original price of the game system?

Worksheet
Profit and loss
MAT08NAWK10058

Technology worksheet
Profit and loss
MAT08NACT10014

Puzzle sheet
Profit and loss
MAT08NAPS00041

Maths clip
Business maths
MAT08NAMC00006

6-10 Profit, loss and GST

Profit and loss

Retailers (store owners) buy goods and sell them to people.

- **Cost price** is how much they buy the item for
- **Selling price** is how much they sell them for
- If they sell the item for **more** than what they paid for it, they make a **profit**
- If they sell the item for **less** than what they paid for it, they make a **loss**

The **percentage profit** or **percentage loss** is usually calculated as a percentage of the cost price.

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Example 24

The Book Bin buys books for $18 each and sells them for $22.50. For each book, calculate:
a the profit
b the percentage profit.

Solution

a Cost price = $18
Selling price = $22.50
Profit = $22.50 − $18 — selling price − cost price
= $4.50

b Percentage profit $= \frac{\$4.50}{\$18} \times 100\%$ — $\frac{\text{profit}}{\text{cost price}} \times 100\%$
$= 25\%$

Example 25

Renae buys a car for $17 500 and sells it 2 years later for $15 000. Find the percentage loss correct to one decimal place.

Solution

Cost price = $17 500
Selling price = $15 000

Loss = $17 500 − $15 000 — cost price − selling price
= $2500

Percentage loss $= \frac{\$2500}{\$17\,500} \times 100\%$ — $\frac{\text{loss}}{\text{cost price}} \times 100\%$
$= 14.2857\ldots\%$
$\approx 14.3\%$

Example 26

Cooper buys a mountain bike for $850 and sells it a year later at 13% profit.
Find the selling price.

Solution

Profit $= 13\% \times \$850$
$= \$110.50$
Selling price $= \$850 + \$110.50$
$= \$960.50$
OR Selling price $= (100\% + 13\%) \times \$850$ Increasing \$850 by 13%.
$= 113\% \times \$750$
$= \$960.50$

Video tutorial
GST
MAT08NAVT10012

GST

Goods and Services Tax (GST) is a tax paid to the government on most goods (items) and services that we purchase. In Australia, GST is charged at 10% of the original price and is generally included in the marked price of the good or service.

Example 27

The selling price of a TV is \$722 + 10% GST. Calculate:
a the GST payable
b the selling price

Solution

a GST $= 10\%$ of \$722
$= \$72.20$

b Selling price $= \$722 + \$72.20$
$= \$794.20$
OR Selling price $= 110\% \times \$722$ Increasing \$722 by 10%.
$= \$794.20$

Example 28

The selling price of a lounge suite is \$2695 with GST included. How much of this price is the GST?

Solution

Selling price + GST $= 100\% + 10\% = 110\%$
110% of the selling price $= \$2695$
10% GST $= \$2695 \div 11$
$= \$245$

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Exercise 6-10 Profit, loss and GST

1 In each situation, state:
- **i** the cost price
- **ii** whether a profit or loss was made.

a A car was bought for $18 700 and sold for $15 000.
b A house was bought for $543 000 and sold for $674 000.
c A computer was sold for $350 when it originally cost $799.
d An antique desk sold for $8000 when it was purchased for $6800.

2 For each situation, find: *See* Example 24
- **i** the profit
- **ii** the percentage profit, correct to one decimal place where necessary.

a cost price $85 selling price $102
b cost price $415 selling price $600
c cost price $2.75 selling price $4.20

3 For each situation, find: *See* Example 25
- **i** the loss
- **ii** the percentage loss, correct to one decimal place where necessary.

a cost price $19 selling price $14
b cost price $2700 selling price $1450
c cost price $79 selling price $58

4 A 3D TV costing $2500 is sold for $3500. Find:
a the profit **b** the percentage profit.

5 A car costing $17 000 is sold for $15 200. Find:
a the loss **b** the percentage loss correct to one decimal place

Worked solutions
Exercise 6-10
MAT08NAWS10054

6 Calculate the selling price for each situation. *See* Example 26

a cost price $200 profit 17%
b cost price $42 profit 70%
c cost price $720 loss 35%
d cost price $1400 loss 9.5%
e cost price $2.50 profit 150%
f cost price $27 loss 40%

7 A block of land is purchased by Erin and Mark for $133 000. Six years later, they sell it for $164 000. Calculate their percentage profit correct to two decimal places.

8 Rose buys a bed for $2700. She later sells it for $1500. Calculate her percentage loss, correct to one decimal place.

9 For each item, calculate the GST payable and the final price. *See* Example 27

a a car priced at $20 900
b a home entertainment unit priced at $1810
c an ice cream priced at $3.00
d a theatre ticket priced at $145
e plumber's fees of $180
f a DVD priced at $27

See Example 28

10 Given that the final price of each item has GST included, calculate the GST and the original price correct to the nearest cent.

a accountant's fees: $792
b piano lessons: $198 per term
c refrigerator: $924
d box of chocolates: $18.70
e batteries: $34.65
f diary: $16.70

Investigation: Uses of percentages

1 Prepare a one-page percentage collage for class display. Look through newspapers, magazines and brochures for examples to cut out. Find photographs or draw situations where percentages are used.
2 Select two examples of situations where percentages are used. Write a set of ten questions for each example, using the rules about percentages you have learnt. Calculate the answers.
3 Swap the questions you wrote in question **2** with others in the class. Each person should answer at least three sets of questions. Ask the student who prepared the questions to mark your answers. If there are any disagreements, check with your teacher.

Worksheet
Percentage problems
MAT08NAWK10059

Homework sheet
Percentages 2
MAT08NAHS10010

Homework sheet
Fractions and percentages revision
MAT08NAHS10011

Quiz
Personal business calculations
MAT08NAQZ00011

6-11 Percentage problems

Exercise 6-11 Percentage problems

1 In a box of 180 oranges, 15% are not ripe. How many oranges are not ripe?

2 A test result is 65 out of 80. What percentage is this?

3 Southside High's hockey team wins 12 games out of 15. What percentage is this?

4 A shirt costing $55 is reduced by 25%. What is the new selling price?

5 In some mixed paint, 25% of the mixture is blue, 15% is yellow and the remainder is white. What percentage is white?

Worked solution
Exercise 6-11
MAT08NAWS10055

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6 Westvale High's basketball team scored 1065 points in a season. Claire shot 22% of the total points. How many points did Claire score?

7 At Upper Darling High, 60% of students are girls. How many boys are there in this school of 870 students?

8 What percentage of pure gold is 9 carat gold, if pure gold is 24 carat?

9 When a floor was being tiled, 830 green tiles were used out of a total of 2075 tiles. What percentage of the floor is made of green tiles?

10 In a kindergarten class of 24 students, there were 3 absent students. Find the percentage of students absent from the class.

11 A real estate agent earns $2\frac{1}{2}$% commission on a sale of $345 000. How much does the agent earn?

Worked solution
Exercise 6-11
MAT08NAWS10055

12 A worker's pay rate of $13.65/hour is increased to $15.20/hour. Find the increase as a percentage of the original pay rate.

13 Find how much rent is paid if it is 30% of a weekly income of $575.

14 Anton receives a 7% pay rise. If his old salary was $67 000 per year, what is his new salary? Select the correct answer **A**, **B**, **C** or **D**.

A $67 007 **B** $67 469 **C** $71 690 **D** $113 900

Technology Weekly budget

In this activity, we will use a spreadsheet to calculate a weekly budget for Abdul, a full-time student with a part-time job in a café.

1. Enter Abdul's weekly expenses into a spreadsheet. Highlight cells B2 to B8, right-click on **Format Cells** and **Currency** with **two decimal places**.

	A	B	C
1	Expenses	Weekly Budget	% of total
2	Rent	157.40	
3	Food	80.00	
4	Fares	34.50	
5	Entertainment	37.00	
6	Mobile phone	11.75	
7	Savings	25.00	
8	Total Expenses		

2. In cell B8, write a formula to calculate the sum of Abdul's weekly expenses.
3. In cell C2, write a formula to calculate the percentage of total expenses spent on rent. Right-click on **Format Cells** and **Number** with **one decimal place**.
4. Select cell C2 and **Fill Down** to cell C8 to copy the formula and calculate the percentage of total expenses belonging to each item.
5. Highlight cells A1 to B7. Click **Insert**, **Pie Chart** to create a pie chart (sector graph) and give your pie chart an appropriate title.
6. Point your mouse (do **not** click) in each sector of the pie chart to read off the percentage breakdown (in brackets) of Abdul's expenses.

Power plus

1 A retailer pays $55 for a chair, then marks it up 60% for the selling price, then adds 10% of this price for GST. Calculate the final selling price.

2 Some items increase in value over time, such as jewellery, antiques and real estate. This is called **appreciation**. A gold chain costs $1500 and appreciates by 8.5% per year. Find:
   a the value of the chain after one year
   b the value of the chain after two years.

3 The Blewes family buys a home for $480 000, and it appreciates at 4% per year. Calculate the home's value:
   a after one year b after two years c after five years

4 a A number is increased by 25%, then the result is decreased by 25%. Is the final answer more than or less than the original number?
   b If a number is increased by 25%, then by what percentage must the result be decreased so that the answer is equal to the original number?

5 Most items lose value over time. This is called **depreciation**. A Ferrari car costs $220 000 and depreciates at 15% per year. Calculate how much the Ferrari is worth:

   a after 1 year b after 2 years c after 5 years.

6 a A number is decreased by $33\frac{1}{3}$%, then the result is increased by $33\frac{1}{3}$%. Is the final answer more than or less than the original number?
   b If a number is decreased by $33\frac{1}{3}$%, then by what percentage must the result be increased so that the answer is equal to the original number?

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Language of maths

commission	fraction	mark-up	quantity
cost price	goods and services tax (GST)	mixed numeral	reciprocal
decimal	improper fraction	numerator	reduction
decrease	increase	percentage	selling price
denominator	loss	profit	simplify
discount	lowest common multiple (LCM)	proper fraction	unitary method

Puzzle sheet
Fractions and percentages find-a-word
MAT08NAPS10018

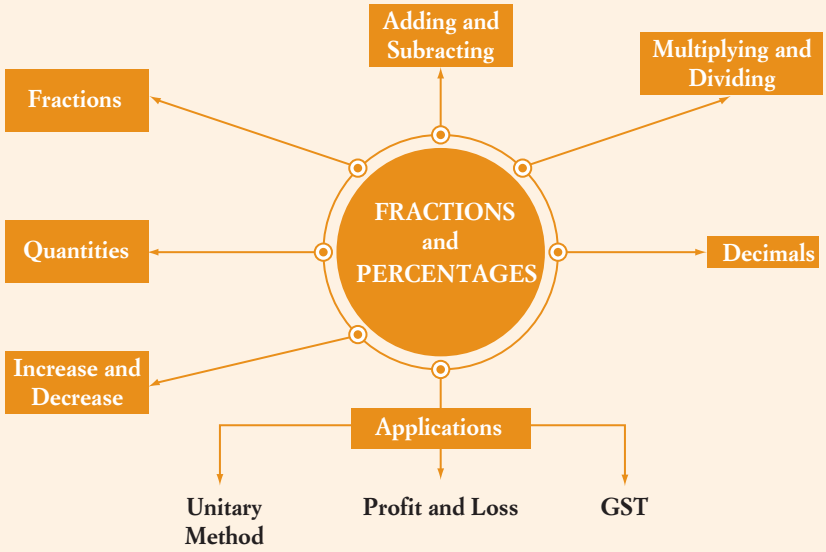
1 Which words in the list above refer to something getting:
   a bigger? **b** smaller?
2 What is the product of a number and its reciprocal?
3 Name one good and one service that would have a **GST**.
4 The 'cent' in percentage means 100. Find other 'cent' words that are related to 100.
5 What do we mean when we say that someone gives '110%' effort or support to something?
6 Copy and complete: "A **loss** is made by a retailer if the **cost price** is _____ than the **selling price**."

Topic overview

- List any rules you remember from your work on fractions and percentages.
- Name at least three jobs where percentages would be used.
- Is there anything you still do not understand about fractions or percentages? See your teacher.
- Give examples of where you may use fractions or percentages in the future.

Worksheet
Mind map: Fractions and percentages
MAT08NAWK10060

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 6 revision

See Exercise 6-01 **1** Simplify each fraction.

a $\frac{6}{8}$ **b** $\frac{28}{48}$ **c** $\frac{35}{45}$

See Exercise 6-01 **2** Convert each improper fraction into a mixed numeral.

a $\frac{15}{4}$ **b** $\frac{22}{5}$ **c** $\frac{26}{6}$

See Exercise 6-01 **3** Convert each mixed numeral to an improper fraction.

a $4\frac{1}{2}$ **b** $3\frac{2}{3}$ **c** $6\frac{4}{5}$

See Exercise 6-01 **4** Arrange these fractions in descending order: $\frac{5}{6}$, $\frac{7}{12}$, $\frac{1}{2}$, $\frac{3}{4}$

See Exercise 6-02 **5** Evaluate each expression.

a $\frac{1}{3}+\frac{2}{5}$ **b** $\frac{4}{7}-\frac{1}{2}$ **c** $\frac{3}{4}+\frac{2}{3}$

d $2\frac{1}{5}+4\frac{1}{2}$ **e** $7-\frac{5}{6}$ **f** $5\frac{3}{8}-2\frac{1}{2}$

See Exercise 6-03 **6** Evaluate each expression.

a $\frac{7}{8}\times\frac{4}{9}$ **b** $\frac{4}{5}\times\frac{2}{3}$ **c** $\frac{6}{7}\times\frac{5}{8}$

d $2\frac{1}{4}\times\frac{3}{10}$ **e** $\frac{1}{6}\div\frac{5}{12}$ **f** $5\frac{3}{8}\div1\frac{1}{2}$

See Exercise 6-04 **7** Convert each percentage into a simplified fraction.

a 74% **b** 29% **c** 65% **d** 157%

See Exercise 6-04 **8** Convert each percentage into a decimal.

a 12% **b** 16.2% **c** 2% **d** 5.4%

See Exercise 6-04 **9** Convert each fraction into a percentage.

a $\frac{7}{8}$ **b** $\frac{12}{50}$ **c** $\frac{2}{9}$ **d** $\frac{33}{40}$

See Exercise 6-04 **10** Convert each decimal into a percentage.

a 0.56 **b** 0.613 **c** 0.7 **d** 0.048

See Exercise 6-04 **11** Arrange these numbers in ascending order: $\frac{3}{4}$, 0.725, 77%, 0.7

See Exercise 6-05 **12** Evaluate each expression.

a 15% of 40 **b** $\frac{1}{4}$ of 64 **c** 27.5% of $880

d $\frac{2}{3}$ of 1 year (in months) **e** 120% of 90 L **f** 35% of 6 m (in cm)

See Exercise 6-06 **13** Gianni scored 68 out of 80 in a maths test. Write his score as:

a a simplified fraction **b** a percentage

See Exercise 6-06 **14** Westvale Christian College has 375 boys and 405 girls. What percentage (correct to one decimal place) of the students are boys?

See Exercise 6-07 **15** A refrigerator with a marked-up price of $545 is discounted by 40%. Calculate its sale price.

See Exercise 6-07 **16** Judy is to receive a salary increase of 3.5%. If she currently earns $65 000, what will her new salary be?

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17 Calculate each expression in your head and write down the answer. *See* Exercise 6-09

a 20% of 55 tonnes **b** $12\frac{1}{2}$% of 96 cm **c** 75% of 12 minutes

18 A deposit of 18% paid on a car is $26 480. What is the cost of the car? *See* Exercise 6-10

19 James bought a guitar for $750, then sold it for $600. Calculate his percentage loss on the cost price. *See* Exercise 6-10

20 The price of a mobile phone is $95.70 including GST. Calculate: *See* Exercise 6-11

a the amount of GST included in this price

b the price of the phone before GST was added

21 Ishween sold $850 worth of books last week. If she receives 5% commission on all sales, calculate her commission. *See* Exercise 6-11